China Petroleum Processing and Petrochemical Technology

Catalyst Research Volume 23, Issue 2, November 2023 Pp. 4991-5001

PERFECT SECOND-DEGREE FUZZY MATCHING FOR FUZZY GRAPH BASED ON VERTICES

Dr. B. Uma Devi [1], J. Anusha [2] R, K. Shanmugha Priya[3]

[1]Associate Professor, Department of Mathematics, S. T. Hindu College, Nagercoil, Affiliated to Manonmanium Sundaranar University, Abishekapati, Tirunelveli.

[2]Research Scholar, Reg no: 18223152092019 ,Department of Mathematics, S. T. Hindu College, Nagercoil, Affiliated to Manonmanium Sundaranar University, Abishekapati, Tirunelveli.

[3]Assistant Professor, Jai Shriram Engineering College, Avinashipalayam, Tamil Nadu.

Abstract

Many real-world problems are represented using Graph theory. Graphs are the models of relations. A graph is an appropriate way of representing information involving relationship between objects where the objects are represented by vertices and the relations are represented by edges. We need to design a fuzzy graph model, when there is vagueness in the description of the objects or in its relationships or in both. In this paper, we introduced perfect **Second-degree** Fuzzy matching for Fuzzy graph based on vertices. We proved the necessary condition under which they are equivalent and also proved that, for a particular condition, a perfect **Second-degree** Fuzzy matching is not a $(\mathbf{2}, \mathbf{k})$ regular Fuzzy graph. We also discussed perfect **Second-degree** Fuzzy matching and **Second-degree** Fuzzy matching number for cycle graph.

Keywords: Fuzzy graph, **Second-degree** Fuzzy matching, perfect **Second-degree** Fuzzy matching, **Second-degree** Fuzzy matching number

1. Introduction

The concept of Fuzzy sets and Fuzzy relations was introduced by L. A.Zadeh [8] in the year 1965. The concept of Fuzzy graph was introduced by Rosenfeld[4] in the year 1975. The concept of regular Fuzzy graphs and regular property ofFuzzy graphs was introduced by Nagoor Gani and Radha[2][3]. New approach on vertex regular Fuzzy graph was introduced by Kailash Kumar Kakkad and Sanjay Sharma[1]. Shakila Banu and Akilandeswari[7] introduced the concept of square perfect Fuzzy matching. Seethalakshmi and Gnanajothi[5] derived the necessary condition for a Fuzzy graph on a cycle. The concept of d_2-degree and total d_2-degree of a vertex in a Fuzzy graph was defined by Sekar and Santhimaheswari[6].

2. Preliminaries

Definition2.1:

A fuzzy graph denoted by $G: (\sigma, \mu)$ on $G^*: (V, E)$ is a pair of functions (σ, μ) where $\sigma: V \to [0,1]$ is a fuzzy subset of a non-empty set V and $\mu: V \, X \, V \to [0, 1]$is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u, v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$ is satisfies, where σ and μ are called membership functions. A fuzzy graph G is complete if $\mu(u, v) = \mu(uv) = \sigma(u) \wedge \sigma(v)$

where uv denotes the edge between u and v. $G^*: (V, E)$ is called the underlying crisp graph of the fuzzy graph $G: (\sigma, \mu)$.

Definition2.2:

Let $G: (\sigma, \mu)$ be a fuzzy graph. The degree of a vertex u is

$$d(u) = \sum_{u \neq v} \mu(uv)$$

since$\mu(uv) > 0$ for$uv \in E$ and $\mu(uv) = 0$ for $uv \notin E$.

The minimum degree of G is $\delta(G) = \wedge \{d(u)/u \in V\}$.

The maximum degree of G is $\Delta(G) = \vee \{d(u)/u \in V\}$.

Definition2.3:

For a given graph G, the d_2 degree of a vertexu in G denoted by $d_2(u)$ means the number of vertices at a distance 2 away from u.

Definition2.4:

For a given fuzzy graph G, the d_2 degree of a Vertex u is

$$d_2(u) = \sum_{\substack{u \neq v \\ u,v \in V}} \mu^2(uv)$$

where,

$\mu^2(uv) = \{\mu(uv_1) \wedge \mu(v_{1,}v)\}$.

Also $\mu(uv) = 0$ for $uv \notin E$.

The minimum $d_2$degree of G is $\delta_2(G) = \wedge \{d_2(u)/u \in V\}$.

The maximum $d_2$degree of G is $\Delta_2(G) = \vee \{d_2(u)/u \in V\}$.

Definition2.5:

A fuzzy graph G is said to be $(2, k)$ regular or d_2 regular if $d_2(u) = k$for all u in G.

3. Perfect Second-degree Fuzzy Matching

Definition 3.1:

Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*: (V, E)$. A subset S of V is called a **Second-degree** fuzzy matching if for each vertex u we have,

$$\sum_{\substack{u \neq v \\ u,v \in V}} \mu^2(uv) \leq \sigma(u)$$

Example 3.1

Let $G:(\sigma,\mu)$ be a fuzzy graph on the cycle$G^*:(V,E)$ where $V=\{v_1,v_2,v_3,v_4,v_5,v_6\}$ and $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$with $e_1=v_1v_2$, $e_2=v_2v_3$, $e_3=v_3v_4$, $e_4=v_4v_5$, $e_5=v_5v_6$, $e_6=v_6v_1$

$\sigma(v_1)=0.4$, $\sigma(v_2)=0.3$, $\sigma(v_3)=0.5$, $\sigma(v_4)=0.8$, $\sigma(v_5)=0.2$, $\sigma(v_6)=0.5$

$\mu(e_1)=0.2$, $\mu(e_2)=0.3$, $\mu(e_3)=0.4$, $\mu(e_4)=0.2$, $\mu(e_5)=0.1$, $\mu(e_6)=0.35$

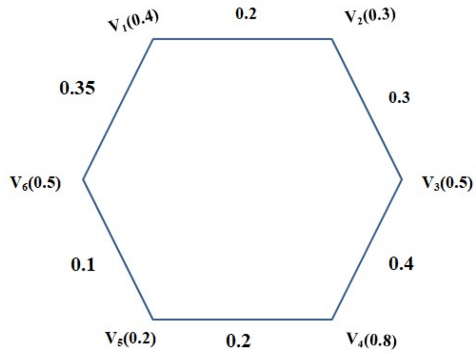

Figure 1: second-degree fuzzy matching

$$\sum_{\substack{v_1\neq v_3\\ v_1,v_3\in V}}\mu^2(v_1v_3)=\mu^2(v_1v_3)+\mu^2(v_1v_5)$$

$$=\{\mu(v_1v_2)\wedge\mu(v_2v_3)\}+\{\mu(v_1v_6)\wedge\mu(v_6v_5)\}$$

$$=\{0.2\wedge 0.3\}+\{0.35\wedge 0.1\}$$

$$=0.2+0.1$$

$$=0.3\leq\sigma(v_1)$$

$$\sum_{\substack{v_2\neq v_4\\ v_2,v_4\in V}}\mu^2(v_2v_4)=0.3+0.2=0.5\nleq\sigma(v_2)$$

$$\sum_{\substack{v_3\neq v_5\\ v_3,v_5\in V}}\mu^2(v_3v_5)=0.2+0.2=0.4\leq\sigma(v_3)$$

$$\sum_{\substack{v_4\neq v_6\\ v_4,v_6\in V}}\mu^2(v_4v_6)=0.1+0.3=0.4\leq\sigma(v_4)$$

$$\sum_{\substack{v_5\neq v_1\\ v_5,v_1\in V}}\mu^2(v_5v_1)=0.1+0.2=0.3\nleq\sigma(v_5)$$

$$\sum_{\substack{v_6\neq v_2\\ v_6,v_2\in V}}\mu^2(v_6v_2)=0.2+0.1=0.3\leq\sigma(v_6)$$

Thus $S = \{v_1, v_3, v_4, v_6\}$ is a $Second$-degree Fuzzy matching in G.

Definition 3.2:

A **Second-degree** Fuzzy matching S is called a Perfect **Second-degree** fuzzy matching if,

$$\sum_{\substack{u \neq v \\ u,v \in V}} \mu^2(uv) = \sigma(u)$$

Definition 3.3:

Let $G: (\sigma, \mu)$ be a Fuzzy graph and S be a Second-degree fuzzy matching. Then **Second-degree** fuzzy matching number $\Gamma(G)$ is defined to be

$$\Gamma(G) = \sum_{u \epsilon S} \mu^2(uv)$$

Example 3.2

In this examplewe have considered Figure 1 and Example 3.1.

$$\Gamma(G) = \sum_{u \epsilon S} \mu^2(uv)$$

where $S = \{v_1, v_3, v_4, v_6\}$

$$\Gamma(G) = \mu^2(v_1 v_3) + \mu^2(v_3 v_5) + \mu^2(v_4 v_6) + \mu^2(v_6 v_2)$$

$$= 0.3 + 0.4 + 0.4 + 0.3 = 1.4$$

Theorem3.1:

Let $G: (\sigma, \mu)$ be a Fuzzy graph on the cycle $G^*: (V, E)$. Then edges of the Fuzzy graph of G is half of their vertices iff, all the vertices of G are perfect **Second-degree** Fuzzy matching and is equivalent to $(2, k)$ is regular fuzzy graph

Proof:

Suppose that σ is a constant function.

Let $\sigma(u) = k$ is a constant for all $u \epsilon V$ and $(uv) = \frac{k}{2}$, for all $(uv) \epsilon E$

Assume that G is a $(2, k)$ regular Fuzzy graph on the cycle $G^*: (V, E)$.

Then $d_2(u) = k$

By definition of d_2 degree of a vertex in Fuzzy graph

$$d_2(u) = \sum_{\substack{u \neq v \\ u,v \in V}} \mu^2(uv)$$

That is,

Catalyst Research Volume 23, Issue 2, November 2023 Pp. 4991-5001

$$\sum_{\substack{u \neq v \\ u,v \in V}} \mu^2(uv) = k$$

Since G is a $(2, k)$ regular Fuzzy graph

$$\Rightarrow \sum_{\substack{u \neq v \\ u,v \in V}} \mu^2(uv) = \sigma(\mu)$$

Therefore Each vertex of u satisfies the Perfect second-degree Fuzzy matching in G.

Now suppose that V is a perfect $second$-degree Fuzzy matching in G.

Since G is a Fuzzy graph on the cycle and only two edges are incident with each vertex for cycles.

For any vertex $u \epsilon V$

$$\Rightarrow \sum_{\substack{u \neq v \\ u,v \in V}} \mu^2(uv) = \sigma(u)$$

By definition,

$$d_2(u) = \sum_{\substack{u \neq v \\ u,v \in V}} \mu^2(uv)$$

$$\Rightarrow d_2(u) = \sigma\,(u)$$

$$\Rightarrow d_2\,(u) = k$$

Hence$G$ is $(2, k)$ regular Fuzzy graph on the cycle.

The converse of the theorem is trivially true.

Example 3.3

China Petroleum Processing and Petrochemical Technology

Catalyst Research Volume 23, Issue 2, November 2023 Pp. 4991-5001

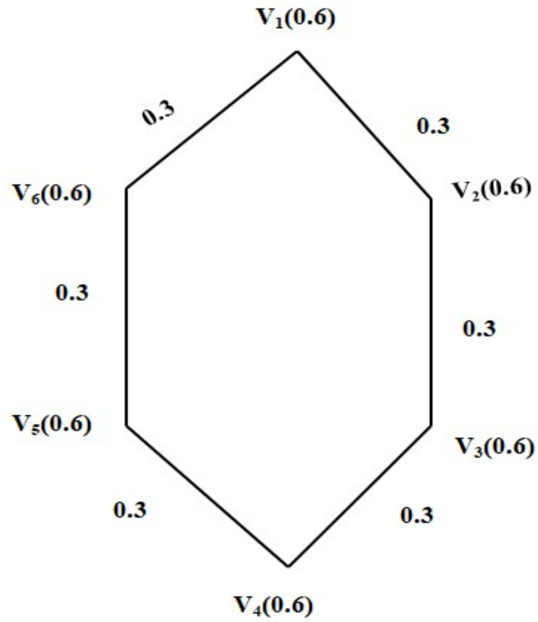

Figure 2:(2,0.6)- regular fuzzy graph

Let $G:(\sigma,\mu)$ be a Fuzzy graph on the cycle $G^*:(V,E)$ where $V=\{v_1,v_2,v_3,v_4,v_5,v_6\}$ and $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$ with $e_1=v_1v_2$, $e_2=v_2v_3$, $e_3=v_3v_4$, $e_4=v_4v_5$,

$e_5=v_5v_6$, $e_6=v_6v_1$

$\sigma(v_1)$ = 0.6, $\sigma(v_2)$ = 0.6, $\sigma(v_3)$ = 0.6, $\sigma(v_4)$ = 0.6, $\sigma(v_5)$ = 0.6 , $\sigma(v_6)$ = 0.6

$\mu(e_1)$ =0.3, $\mu(e_2)$ =0.3, $\mu(e_3)$=0.3, $\mu(e_4)$=0.3, $\mu(e_5)$=0.3, $\mu(e_6)$=0.3

$$\sum_{\substack{v_1\neq v_3\\ v_1,v_3\in V}}\mu^2(v_1v_3)=\mu^2(v_1v_3)+\mu^2(v_1v_5)$$

$=\{\mu(v_1\,v_2)\wedge\mu(v_2v_3)\}+\{\mu(v_1\,v_6)\wedge\mu(v_6v_5)\}$

$=\{0.2\wedge 0.2\}+\{0.2\wedge 0.2\}$

$=0.3+0.3$

$=0.6=\sigma(v_1)$

Similarly,

$$\sum_{\substack{v_2\neq v_4\\ v_2,v_4\in V}}\mu^2(v_2v_4)=0.3+0.3=0.6=\sigma(v_2)$$

$$\sum_{\substack{v_3\neq v_5\\ v_3,v_5\in V}}\mu^2(v_3v_5)=0.3+0.3=0.6=\sigma(v_3)$$

$$\sum_{\substack{v_4 \neq v_6 \\ v_4, v_6 \in V}} \mu^2(v_4 v_6) = 0.3 + 0.3 = 0.6 = \sigma(v_4)$$

$$\sum_{\substack{v_5 \neq v_1 \\ v_5, v_1 \in V}} \mu^2(v_5 v_1) = 0.3 + 0.3 = 0.6 = \sigma(v_5)$$

$$\sum_{\substack{v_6 \neq v_2 \\ v_6, v_2 \in V}} \mu^2(v_6 v_2) = 0.3 + 0.3 = 0.6 = \sigma(v_6)$$

Hence G is a perfect$Second$-degree Fuzzy matching and also $(2, 0.6)$ regular Fuzzy graph.

Theorem3.2:

Let $G: (\sigma, \mu)$ be a $(2, k)$ regular Fuzzy graph on the cycle $G^*: (V, E)$. $\sigma(u)$ and $\mu(uv)$are constant functions where $\mu(uv) \leq \sigma(u)$and $\mu(uv) \neq \frac{1}{2}\sigma(u)$ for all $(uv) \epsilon E$ then V is not a perfect **Second-degree** Fuzzy matching of G.

Proof:

Let $\sigma(u)$ = k and $\mu(uv) = c$ are constant functions for all $u, v \epsilon V$ where $c \leq k$ and $c \neq k/2$

To prove V is not a perfect **Second-degree** Fuzzy matching of G.

Suppose that G is a Fuzzy graph on the cycle and only two edges are incident with each vertex for the cycles.

For any vertex $u \epsilon V$

$$\Rightarrow \sum_{\substack{u \neq w \\ u, w \in V}} \mu^2(uw) = \mu^2(uw) + \mu^2(uv)$$

$$= c + c$$

$$= 2c \leq k$$

$$= c \leq k/2 \text{ but } c \neq k/2$$

$$= c < k/2$$

$$\Rightarrow \sum_{\substack{u \neq w \\ u, w \in V}} \mu^2(uw) \neq k$$

$$\Rightarrow \sum_{\substack{u \neq w \\ u, w \in V}} \mu^2(uw) \neq \sigma(u)$$

Catalyst Research Volume 23, Issue 2, November 2023 Pp. 4991-5001

Hence V is not a perfect **Second-degree** Fuzzy matching of G.

Example 3.4

Let $G:(\sigma,\mu)$ be a Fuzzy graph on the cycle $G^*:(V,E)$where $V = \{v_1, v_2, v_3, v_4\}$and $E = \{e_1, e_2, e_3, e_4\}$ with $e_1 = v_1v_2$, $e_2 = v_2v_3$, $e_3 = v_3v_4$, $e_4 = v_4v_1$

$\sigma(v_1) = 0.6$, $\sigma(v_2) = 0.6$, $\sigma(v_3) = 0.6$, $\sigma(v_4) = 0.6$

$\mu(e_1) = 0.2$, $\mu(e_2) = 0.2$, $\mu(e_3) = 0.2$, $\mu(e_4) = 0.2$

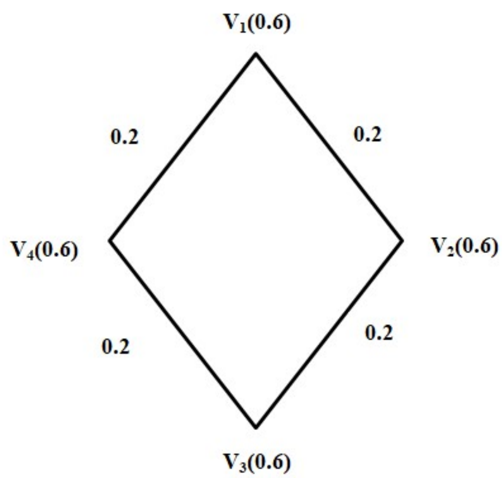

Figure 3: Not a perfect **Second-degree** fuzzy matching

For any vertex $u \in V$ we have

$$\sum_{\substack{u \neq v \\ u,v \in V}} \mu(uv) = \sigma(u)$$

$$\sum_{\substack{v_1 \neq v_2 \\ v_1, v_2 \in V}} \mu(v_1v_2) = 0.2 + 0.2 = 0.4 \neq \sigma(v_1)$$

$$\sum_{\substack{v_2 \neq v_3 \\ v_2, v_3 \in V}} \mu(v_2v_3) = 0.2 + 0.2 = 0.4 \neq \sigma(v_2)$$

$$\sum_{\substack{v_3 \neq v_4 \\ v_3, v_4 \in V}} \mu(v_3v_4) = 0.2 + 0.2 = 0.4 \neq \sigma(v_3)$$

$$\sum_{\substack{v_4 \neq v_1 \\ v_4, v_1 \in V}} \mu(v_4v_1) = 0.2 + 0.2 = 0.4 \neq \sigma(v_4)$$

Hence G is not a Perfect **Second-degree** Fuzzy matching.

Theorem3.3:

Let G be a Perfect **Second-degree** Fuzzy matching on the cycle $G^*:(V,E)$ of length $n \geq 5$. If

$$\sigma(u_i)=\begin{cases} k/4 & , i=1,2,n-1,n \\ k/3 & , i=3,4,\ldots n-2 \\ \text{for all } u\in V \end{cases}$$

and

$$\mu(e_i)=\begin{cases} k/6 & , i=1,2,3,4,\ldots n-1 \\ k/12 & , i=n \\ \text{for all}(uv)\in E \end{cases}$$

Then G is not $(2,k)$ regular Fuzzy graph.

Proof:

Let $G:(\sigma,u)$ be a Perfect **Second-degree** Fuzzy matching on the Cycle $G^*:(V,E)$is any length ≥ 5.

To prove that G is not a $(2,k)$ regular Fuzzy graph.

Let $v_1,v_2,v_3,\ldots\ldots.v_n$ be the vertices and $e_1,e_2,\ldots\ldots\ e_n$ be edges of a cycle onG^* in that order.

By definition,

$$d_2(u)=\sum_{\substack{u\neq v\\ u,v\in V}}\mu^2(uv)$$

$$d_2(v_1)=\sum_{\substack{v_1\neq v_3\\ v_1,v_3\in V}}\mu^2(v_1v_3)$$

$$=\mu^2(v_1v_3)+\mu^2(v_1v_{n-1})$$

$$=\{\mu(v_1v_2)\wedge\mu(v_2v_3)\}+\{\mu(v_1v_n)\wedge\mu(v_nv_{n-1})\}$$

$$=\{\mu(e_1)\wedge\mu(e_2)\}+\{\mu(e_n)\wedge\mu(e_{n-1})\}$$

$$=\{k/6\wedge k/6\}+\{k/12\wedge k/6\}$$

$$=k/6+k/12=k/4=\sigma(v_1)$$

$$d_2(v_2)=\{\mu(e_2)\wedge\mu(e_3)\}+\{\mu(e_1)\wedge\mu(e_n)\}$$

$$=\{k/6\wedge k/6\}+\{k/6,\wedge k/12\}$$

$$=k/6+k/12=k/4=\sigma(v_2)$$

$$d(v_3)=\{\mu(e_3)\wedge\mu(e_4)\}+\{\mu(e_2)\wedge\mu(e_1)\}$$

$=\{{}^{k}/_{6} \wedge {}^{k}/_{6}\} + \{{}^{k}/_{6} \wedge {}^{k}/_{6}\}$

$= {}^{k}/_{6} + {}^{k}/_{6} = {}^{k}/_{3} = \sigma(v_3)$

- - - - -

- - - - -

- - - - -

$d_2(v_i) = \{\mu(e_i) \wedge \mu(e_{i+1})\} + \{\mu(e_{i-1}) \wedge \mu(e_{i-2})\}$

$= {}^{k}/_{6} + {}^{k}/_{6} = {}^{k}/_{3} = \sigma(v_i)$

- - - -

- - - - -

- - - - -

h

$d_2(v_{n-1}) = \{\mu(e_{n-1}) \wedge \mu(e_n)\} + \{\mu(e_{n-2}) \wedge \mu(e_{n-3})\}$

$= \{{}^{k}/_{6} \wedge {}^{k}/_{12}\} + \{{}^{k}/_{6}, \wedge {}^{k}/_{6}\}$

$= {}^{k}/_{12} + {}^{k}/_{6} = {}^{k}/_{4} = \sigma(v_{n-1})$

$d_2(v_n) = \{\mu(e_{n-1}) \wedge \mu(e_n)\} + \{\mu(e_{n-2}) \wedge \mu(e_{n-3})\}$

$= \{{}^{k}/_{6} \wedge {}^{k}/_{12}\} + \{{}^{k}/_{6} \wedge {}^{k}/_{6}\} = {}^{k}/_{12} + {}^{k}/_{6} = {}^{k}/_{4} = \sigma(v_n)$

Hence G is not a$(2, k)$ regular Fuzzy graph

Conclusion

In this paper, we have introduced perfect **Second-degree** Fuzzy matching for Fuzzy graph based on vertices on the cycle. We proved the necessary condition under which they are equivalent and also proved that, for a particular condition, a perfect **Second-degree** Fuzzy matching is not a $(\mathbf{2}, \mathbf{k})$regular Fuzzy graph. We also discussed perfect **Second-degree** Fuzzy matching and **Second-degree** Fuzzy matching number for cycle graph. For the future work we can extend to complete graph regular graph and finally for any connected graph.

References

1. Kailash Kumar Kakkad and Sanjay Sharma, New approach on Regular Fuzzy graph, Global Journal of Pure and Applied Mathematics, Volume 13, 2017, 3753 – 3766

2. NagoorGani A. and Radha K., On Regular Fuzzy graphs, Journal of Physical Science, Volume 12 ,2008, 33 - 40
3. NagoorGani A. and Radha K., Regular property of Fuzzy graphs, Bulletin of Pure and Applied Sciences, Volume 27E (No.2), 2008, 415 – 423
4. Rosenfeld A.,L. A. Zadek, K. S. Fu and M. Shimura, Fuzzy Graphs, Fuzzy Sets and their Applications, Academic Press, 1975, 77 - 95
5. Seethalakshmi R. and Gnanajothi R. B., A note on Perfect Fuzzy matching International Journal of Pure and Applied Mathematics Volume 94(2), 2014, 155 – 161
6. Sekar C. and SanthiMaheswari R., On (2, k) – regular and totally (2, k) regular Fuzzy graphs, International Journal of Mathematics and Soft computing, Volume 4, No. 2, 2014, 59 – 69
7. Shakila Banu P. and Akilandeswari R., Square Perfect Fuzzy matching, Inter J. Fuzzy Mathematical Archive 11(1), 2016, 45 – 52
8. Zadeh L. A., Fuzzy Sets*, Informationand control, 8, 1965, 338 – 353