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STRONGLY EDGE MULTIPLICATIVE LABELING OF SOME GRAPHS

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Abstract

A graph $G = (V, E)$ with p vertices and q edges is said to be strongly edge multiplicative if the edges of G can be labeled with distinct integers from $1,2, \dots, q$ such that labels induced on the vertices obtained by the product of the labels of incident edges are distinct. We discuss strongly edge multiplicative labeling of some graphs. Also using the definition of strongly edge multiplicative labeling we define strongly vertex number of graphs. Let G be the strongly edge multiplicative graph and f^* be the corresponding induced labeling of G.We introduce the new notation strongly induced vertex number $\gamma_v(G) = \max\{f^*(v): v\epsilon G\}$.

Keywords: Strongly Edge multiplicative Labeling, Induced vertex Number, Path graph, Star graph, Fan graph.

AMS Subject Classifications:05C78.

Introduction

The assignments of values typically represented by integers using several appropriate mathematical rules to the vertices and / or edges of the given graph is termed as graph labeling. Formally, in 1967 Alex Rosa[1] introduced the concept of graph labeling. There is an enormous literature regarding labeling of many familiar classes of graphs.In this paper we deal only finite, simple, connected and undirected graphs. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$. Hence $|V(G)|$ and $|E(G)|$ are the number of vertices and number of edges of G respectively.

In 2001, Beineke and Hegde introduced multiplicative labeling and proved that every graph admits a multiplicative labeling. The Strongly multiplicative labeling was introduced by Beineke and Hegde. Now strongly multiplicative labeled graphs often serve as models in a wide range of applications. Such applications including coding theory and communication network . In 2020 Jeyabalan and Kumar introduce the concept Strongly edge multiplicative graphs. A graph $G = (V, E)$ with p vertices and q edges is said to be strongly edge multiplicative if the edges of G can be labeled with distinct integers from $1,2, \dots, q$ such that labels induced on the vertices obtained by the product of the labels of incident edges are distinct. We investigate some properties of strongly edge multiplicative graph and generalized book graph. Let G be the strongly edge multiplicative graph and f^* be the corresponding induced labeling of G. We introduce the new

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notation strongly induced vertex number $\gamma_v(G) = \max\{f^*(v): v \epsilon G\}$. We discuss Path graph, Cycle graph, Star graph, Fan graph, Bistar graph.

1 Preliminaries

Definition:1.1 [5] A walk in which no vertex is repeated is called a path P_n. It has n vertices and $n-1$ edges.

Definition 2.4. [15] The vertex set of L(G) is in one-to-one correspondence with the edge set of G and two vertices of L(G)are joined by an edge if and only if the corresponding edges of G are adjacent in G. The graph L(G) is called the line graph or the edge graph of G.

Definition:1.33 [12]The triangular book with n-pages is defined as n copies of cycle C_3 sharing a common edge. The common edge is called the spine or base if the book. This graph denoted by $B(3, n)$.

We construct the generalized book with n-pages is defined a n copies of cycle C_m sharing a common edge.The common edge is called the spine or base of the book.The graph denoted by $B(m, n)$.

Definition:1.4 [14] The star graph S_n is a complete bipartite graph $K_{1,n}$ where n represents the number of vertices and S_n has n-1 edges.

Definition:1.5 [14] The bistar $B_{(m,n)}$ is a graph obtained from K_2 by joining m pendant edges to each end of K_2.The edge K_2 is called the central edge $B_{(m,n)}$ and the vertices of K_2 are called the central vertices of $B_{(m,n)}$.

Definition:1.5[16] The fan fraph f_n is obtained by taking n-3 concurrebt chords in a cycle C_n .The vertex at which all chords are concurrent is called the apex vertex.

2. Properties of strongly edge multiplicative graph.

Theorem 2.1 Let G_1 and G_2 be the strongly edge multiplicative graph. Then $G_1 \cup G_2$ is also a strongly edge multiplicative graph.

Proof: Let G_1 and G_2 be two strongly edge multiplicative graphs with number of edges m_1 and m_2 respectively. Then the graph $G_1 \cup G_2$ will have $m_1 + m_2$ edges. Since G_1 is strongly edge multiplicative the induced labeling of the vertices are distinct. Suppose the edges of G_2 are labeled by $1,2, \dots, m_2$ then relabel the corresponding edges by $m_1 + 1, m_2 + 2, \dots, m_1 + m_2$ respectively. As edges are relabeled the induced vertices are all distinct in G_2.Hence $G_1 \cup G_2$ is a strongly edge multiplicative graph.

Remark 2.2 If $G_1 \cup G_2$ is strongly edge multiplicative, then G_1 and G_2 neednot be strongly edge multiplicative graph.

Example 2.3: Let $G_1 \cup G_2 = P_3 \cup C_n$.Consider G_1 be the path P_3 and G_2 be the cycle graph C_n.Clearly $G_1 \cup G_2$ is a strongly edge multiplicative graph. In [edge ref] proved that C_n is a strongly edge multiplicative graph and P_3 is not strongly edge multiplicative graph.

Theorem 2.4 Let G_1 and G_2 be the two strongly edge multiplicative graph connected by an edge.Then the resultant graph is a strongly edge multiplicative graph.

Proof: Let G_1 and G_2 be two strongly edge multiplicative graphs with number of edges $m_1$and m_2 respectively. Then the graph$G_1 \cup G_2$ will have $m_1 + m_2$ edges. Consider G be the resultant graph with vertex se $V(G) = V(G_1) \cup V(G_2)$ and $E(G) = E(G_1 \cup G_2) \cup \{e\}$ be the edge set of G. Now we label the edge e is 1 where $e = u_i v_i$, $u_i \epsilon G_1$, $v_i \epsilon G_2$ and the remaining edges G_1 and G_2 are labeled by $2,3, \dots, m_1 + 1$ and $m_1 + 2, m_1 + 3, \dots, m_1 + m_2 + 1$.Hence G is a strongly edge multiplicative graph.

3. Results on Strongly Edge Multiplicative Graphs

Theorem 3.1. Let G be a graph obtained by connecting two cycles C_k and C_m to a path of length t. Then G is a strongly edge multiplicative graph for all $k, m \geq 3$ and $t \geq 2$.

Proof. Let G be a graph with vertex set $V(G) = \{u_p, v_q, w_r: 1 \leq p \leq k, 1 \leq q \leq t-1, 1 \leq r \leq m$, where $u_1, u_2, \dots, u_k$ be the vertices of the cycle C_k, $w_1, w_2, \dots, w_m$ be the vertices of cycle C_m and $u_1, v_1, v_2, \dots, v_{t-1}, w_1$ be the vertices of path of length t.Let $E(G) = \{u_i u_{i+1}: 1 \leq p \leq k - 1\} \cup \{u_k u_1, w_m w_1\} \cup \{u_1 v_1, v_{t-1} w_1\} \cup \{v_i v_{i+1}: 1 \leq q \leq t-2\} \cup \{w_i w_{i+1}: 1 \leq r \leq m-1\}$ be the edge set of G.

We note that $|V(G)| = k + m + t - 1$ and $|E(G)| = k + m + t$

We define $f: E(G) \rightarrow \{1,2,3, \dots, k + m + t\}$ as follows.

$$f(u_k u_1) = k, \quad f(u_1 v_1) = k+1, \quad f(v_{t-1} w_1) = k + t$$
$$f(u_p u_{p+1}) = i, \qquad 1 \leq p \leq k$$
$$f(v_q v_{q+1}) = k + q + 1, \qquad 1 \leq q \leq k$$
$$f(w_r w_{r+1}) = k + t + r, \qquad 1 \leq r \leq k$$
$$f(w_m w_1) = k + m + t$$

Clearly the edge labels are distinct.

The induced vertex function $f^*: V(G) \rightarrow N$ defined by

$$f^*(u_1) = km$$
$$f^*(w_1) = (k+t)(k+t+1)(k+m+t)$$
$$f^*(u_{p+1}) = p(p+1), \qquad 1 \leq p \leq k-1$$
$$f^*(v_q) = (k+q)(k+q+1), \qquad 1 \leq q \leq t-1$$
$$f^*(w_{r+1}) = (k+t+r)(k+t+r+1), \qquad 1 \leq r \leq m-1$$

Clearly vertex labels are distinct.

Thus the labeling pattern defined above satisfies the vertex conditions and edge conditions of strongly edge multiplicative labeling.

Hence G a is strongly edge multiplicative graph.

Illustration: The strongly edge multiplicative labeling of G,k=4,m=5,t=3

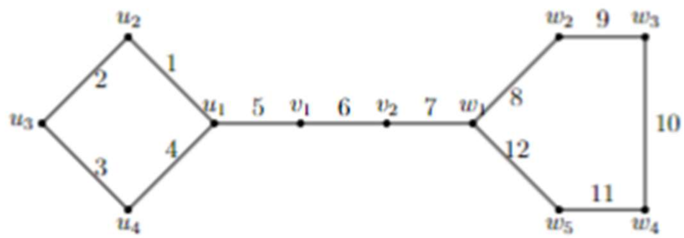

Figure :1

Theorem 2.2. The line graph $L(C_n)$ is a strongly edge multiplicative graph for all $n \geq 3$.

Proof. Let $G = L(C_n)$ be the line graph of cycle C_n with vertex set $V(G) = \{v_i : 1 \leq i \leq n\}$ and edge set $E(G) = \{v_1v_2, v_nv_{n-1}\} \cup \{v_iv_{2i+1} : 1 \leq i \leq n \quad i \text{ is odd}\} \cup \{v_iv_{2i} : 1 \leq i \leq n \quad i \text{ is even}\}$.

We note that $|V(G)| = n$ and $|E(G)| = n$

We define $f: E(G) \to \{1,2,3, \dots, n\}$ as follows.

$$f(v_1v_2) = 1$$

$$f(v_nv_{n-1}) = n$$

$$f(v_iv_{2i+1}) = 2i, \qquad 1 \leq i \leq n \quad i \text{ is odd}$$

$$f(v_iv_{2i}) = 2i + 1, \qquad 1 \leq i \leq n \quad i \text{ is even}$$

Clearly the edge labels are distinct.

The induced vertex function $f^*: V(G) \to N$ defined by

$$f^*(v_i) = (i-1)(i+1), \qquad 1 \leq i \leq n \quad i \text{ is odd}$$

$$f^*(v_i) = (i-1)(i+1), \qquad 1 \leq i \leq n \quad i \text{ is even}$$

Clearly vertex labels are distinct.

Thus the labeling pattern defined above satisfies the vertex conditions and edge conditions of strongly edge multiplicative labeling.

Hence G a is strongly edge multiplicative graph.

Illustration: The strongly edge multiplicative labeling of C_7

Figure 2

Theorem 2.3. The generalized book graph $B(m,n)$ is a strongly edge multiplicative graph for all $n \text{ and } m \geq 3$.

Proof. Let $B(m,n)$ be the generalized book graph with vertex set $V(B(m,n)) = \{v_0, v_0', v_{ij} : 1 \leq i \leq n, \ 1 \leq j \leq m-2\}$,where v_0, v_0' be the common vertices of the n-copies of cycle C_m and edge set $E(B(m,n)) = \{v_0v_0'\} \cup \{v_0v_{i1} : 1 \leq i \leq n\} \cup \{v_0'v_{i(m-2)} : 1 \leq i \leq n\} \cup \{v_{ij}v_{i(j+1)} : 1 \leq i \leq n, \ 1 \leq j \leq m-3\}$..

We note that $|V(B(m,n))| = mn - 2n + 2$ and $|E(B(m,n))| = n(m-1) + 1$

We define $f: E(B(m,n)) \rightarrow \{1,2,3,\dots,n(m-1)+1\}$ as follows.

$$f(v_0 v_{i1}) = m(i-1)-(i-1)+2, \quad 1 \le i \le n$$
$$f\left(v_0' v_{i(m-2)}\right) = mi-(i-1), \quad 1 \le i \le n$$
$$f\left(v_{ij} v_{i(j+1)}\right) = m(i-1)-i+3+j, \quad 1 \le i \le n, \ 1 \le j \le m-3$$

Clearly the edge labels are distinct.

The induced vertex function $f^*: V(B(m,n)) \rightarrow N$ defined by

$$f^*\left(v_{ij}\right) = [m(i-1)-i+3+j][m(i-1)-i+2+j], \quad 1 \le i \le n, \ 1 \le j \le m-2$$
$$f^*(v_0) = \prod_{i=1}^{n}[m(i-1)-(i-1)+2]$$
$$f^*(v_0') = \prod_{i=1}^{n}[mi-(i-1)]$$

Clearly vertex labels are distinct.

Thus the labeling pattern defined above satisfies the vertex conditions and edge conditions of strongly edge multiplicative labeling.

Hence $B(m,n)$ a is strongly edge multiplicative graph.

Illustration : The strongly edge multiplicative labeling of $B(5,3)$

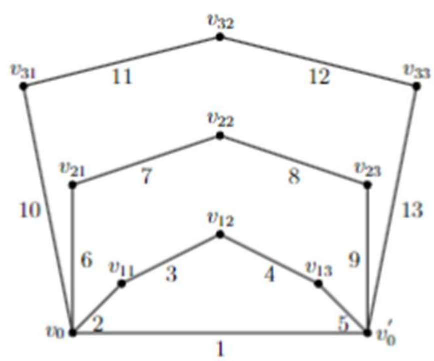

Figure 3

3. Strongly Induced Vertex Number

Definition: Let G be the strongly edge multiplicative graph and f^* be the corresponding induced labeling of G. We introduce the new notation strongly induced vertex number $\gamma_v(G) = \max\{f^*(v): v \epsilon G\}$.

Theorem.3.1. Let G and $G_1 \cup G_2$ be two strongly edge multiplicative graph. Then $\gamma_v(G) \le \gamma_v(G_1 \cup G_2)$.

Proof: By the definition union of graph and strongly multiplicative graph. Clearly $\gamma_v(G) \le \gamma_v(G_1 \cup G_2)$.

Theorem.3.2. For any path P_n, $\gamma_v(P_n) = (n-1)(n-2)$.

Proof: In[7] proved that P_n is strongly edge multiplicative graph. The edges are labeled by $\{1,2,\dots,(n-1)\}$ and the induced vertex labels are $\{1,2,\dots,(n-1)(n-2)\}$.Clearly $\gamma_v(P_n) = (n-1)(n-2)$.

Theorem.3.2. For any star S_n, $\gamma_v(S_n) = n!$.

Proof: In[7] proved that S_n is s strongly edge multiplicative graph. The edges are labeled by $\{1,2,\dots,n\}$ and the induced vertex labels are $\{1,2,\dots,n,n!\}$.Clearly $\gamma_v(S_n) = n!$.

Theorem.3.2. For any bistar $B_{(m,n)}$, $\gamma_v\left(B_{(m,n)}\right) = \prod(n+2, n+3, \dots, 2n+1)$.

Proof: In[7] proved that $B_{(m,n)}$is a strongly edge multiplicative graph. The edges are labeled by $\{1,2,\dots,(2n+1)\}$ and the induced vertex labels are $\{(1,2,\dots,(2n+1), \prod\left(1,2,\dots,(n+1)\right), \prod(n+2,\dots,(2n+1))\}$.Clearly $\gamma_v(B_{(m,n)}) = \prod(n+2, n+3, \dots, 2n+1)$.

Theorem.3.2. For any fan F_n, $\gamma_v(F_n) = (2n)(3n)$.

Proof: In[7] proved that F_n is a strongly edge multiplicative graph. The edges are labeled by $\{1,2,\dots,3n\}$ and the induced vertex labels are $\{1(2n+1), 2(2n+1), 3(2n+2), 4(2n+2), \dots, (2n-1)(3n), (2n)(3n)\}$. Clearly $\gamma_v(F_n) = (2n)(3n)$.

Conclusion

In this paper, we discussed the strongly edge multiplicative labeling. The strongly edge multiplicative labeling conditions are satisfied the some classes of graphs. There may be many interesting strongly edge multiplicative graphs can be constructed in future

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