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Intuitionistic Fuzzy $\widehat{g}^*$ Semi Open Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract: This article is intended to launch another breakthrough in ***Intuitionistic Fuzzy*** $\widehat{g}^*$***Semi Closed sets*** namely ***Intuitionistic Fuzzy*** $\widehat{g}^*$***Semi Open Mappings.*** We also poster some essential comparative notions with other closed mappings and engage into a deeper analysis of their characterizations.

Key Words: Intuitionistic Fuzzy $\widehat{g}^*$ Semi Open set ($\mathcal{IF}\widehat{g}^*s\mathcal{OS}$), Intuitionistic Fuzzy $\widehat{g}^*$ Semi Closed Mapping ($\mathcal{IF}\widehat{g}^*s\mathcal{CM}$) and Intuitionistic Fuzzy $\widehat{g}^*$ Semi Open Mapping ($\mathcal{IF}\widehat{g}^*s\mathcal{OM}$).

AMS Subject Classifications (2000): 54A40, 03F55

I. INTRODUCTION

Zadeh (1965)[15] with his invention of fuzzy sets began a new page in the history of Mathematics. Chang [2] made it a point to introduce fuzzy topology in 1967. Atanassov [1] led this to another level of generalization by his Intuitionistic Fuzzy Sets in 1986. Coker [3] constructed Intuitionistic Fuzzy Topological spaces. Intuitionistic fuzzy closed mapping was introduced and investigated by Gurcay et al. [6] in 1997. In 2000, Lee et al.[7] investigated the properties of open and closed mappings in intuitionistic fuzzy topological spaces. In recent past Pious Missier, Peter Arokiaraj and et.al [6] introduced Intuitionistic Fuzzy $\widehat{g}^*$ Semi closed sets in Intuitionistic Fuzzy Topological Spaces. Here we proceed to present our findings on ***Intuitionistic Fuzzy*** $\widehat{g}^*$ ***Semi Open Mappings in Intuitionistic Fuzzy Topological Spaces***.

II. PRELIMINARIES

Definition 2.1. [1] Let $\mathbb{U}$ be a universal set. Then $\mathfrak{M}_{if} = \{\langle \mathbb{u}, \mu_{\mathfrak{M}_{if}}(\mathbb{u}), \upsilon_{\mathfrak{M}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$ is called as an intuitionistic fuzzy subset ($\mathcal{IFS}$ in short) in $\mathbb{U}$. Here the functions $\mu_{\mathfrak{M}_{if}}: \mathbb{U} \rightarrow [0,1]$ and $\upsilon_{\mathfrak{M}_{if}}: \mathbb{U} \rightarrow [0,1]$ denote the degree of membership (namely $\mu_{\mathfrak{M}_{if}}(\mathbb{u})$) and the degree of non-membership (namely $\upsilon_{\mathfrak{M}_{if}}(\mathbb{u})$) of each element $\mathbb{u} \in \mathbb{U}$ to the set $\mathfrak{M}_{if}$ respectively and $0 \leq \mu_{\mathfrak{M}_{if}}(\mathbb{u}) + \upsilon_{\mathfrak{M}_{if}}(\mathbb{u}) \leq 1$ for each $\mathbb{u} \in \mathbb{U}$. The set of all $\mathcal{IFS}$s in $\mathbb{U}$ is denoted by $\mathcal{IFS}(\mathbb{U})$. For any two $\mathcal{IFS}$s $\mathfrak{M}_{if}$ and $\mathfrak{N}_{if}$, $(\mathfrak{M}_{if} \cup \mathfrak{N}_{if})^C = \mathfrak{M}_{if}{}^C \cap \mathfrak{N}_{if}{}^C$; $(\mathfrak{M}_{if} \cap \mathfrak{N}_{if})^C = \mathfrak{M}_{if}{}^C \cup \mathfrak{N}_{if}{}^C$.

Definition2.2: [1] If $\mathfrak{M}_{if} = \{\langle \mathbb{u}, \mu_{\mathfrak{M}_{if}}(\mathbb{u}), \upsilon_{\mathfrak{M}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$ and $\mathfrak{N}_{if} = \{\langle \mathbb{u}, \mu_{\mathfrak{N}_{if}}(\mathbb{u}), \upsilon_{\mathfrak{N}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$ be two $\mathcal{IFS}s(\mathbb{U})$, then

(a) $\mathfrak{M}_{if} \subseteq \mathfrak{N}_{if}$ if and only if $\mu_{\mathfrak{M}_{if}}(\mathbb{u}) \leq \mu_{\mathfrak{N}_{if}}(\mathbb{u})$ and $\upsilon_{\mathfrak{M}_{if}}(\mathbb{u}) \geq \upsilon_{\mathfrak{N}_{if}}(\mathbb{u})$ for all $\mathbb{u} \in \mathbb{U}$,

(b) $\mathfrak{M}_{if} = \mathfrak{N}_{if}$ if and only if $\mathfrak{M}_{if} \subseteq \mathfrak{N}_{if}$ and $\mathfrak{M}_{if} \supseteq \mathfrak{N}_{if}$,

(c) $\mathfrak{M}_{if}{}^C = \{\langle \mathbb{u}, \upsilon_{\mathfrak{M}_{if}}(\mathbb{u}), \mu_{\mathfrak{M}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$ (complement of $\mathfrak{M}_{if}$),

(d) $\mathfrak{M}_{if} \cup \mathfrak{N}_{if} = \{\langle \mathbb{u}, \mu_{\mathfrak{M}_{if}}(\mathbb{u}) \vee \mu_{\mathfrak{N}_{if}}(\mathbb{u}), \upsilon_{\mathfrak{M}_{if}}(\mathbb{u}) \wedge \upsilon_{\mathfrak{N}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$,

(e) $\mathfrak{M}_{if} \cap \mathfrak{N}_{if} = \{\langle \mathbb{u}, \mu_{\mathfrak{M}_{if}}(\mathbb{u}) \wedge \mu_{\mathfrak{N}_{if}}(\mathbb{u}), \upsilon_{\mathfrak{M}_{if}}(\mathbb{u}) \vee \upsilon_{\mathfrak{N}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$,

(f) $(\mathfrak{M}_{if} \cup \mathfrak{N}_{if})^C = \mathfrak{M}_{if}{}^C \cap \mathfrak{N}_{if}{}^C$ and $(\mathfrak{M}_{if} \cap \mathfrak{N}_{if})^C = \mathfrak{M}_{if}{}^C \cup \mathfrak{N}_{if}{}^C$.

(h) $\tilde{\mathbf{0}} = \langle \mathbb{u}, 0, 1\rangle$(empty set) and $\tilde{\mathbf{1}} = \langle \mathbb{u}, 1, 0\rangle$ (whole set).

Definition 2.3: [3] An intuitionistic fuzzy topology ($\mathcal{IFT}$) on $\mathbb{U}$ is a family of $\mathcal{IFS}s$ in $\mathbb{U}$, satisfying the following axioms.

1. $\tilde{0}$, $\tilde{1} \in \tau_{if}$
2. $\mathfrak{M}_{if} \cap \mathfrak{N}_{if} \in \tau_{if}$ for any $\mathfrak{M}_{if}$, $\mathfrak{N}_{if} \in \tau_{if}$
3. $\cup \mathfrak{M}_{{if}_i} \in \tau_{if}$ for any family $\{\mathfrak{M}_{{if}_i} / i \in \mathcal{J}\} \subseteq \tau_{if}$.

The pair $(\mathbb{U}, \tau_{if})$ is called an intuitionistic fuzzy topological space ($\mathcal{IFTS}$) and any $\mathcal{IFS}$ in τ_{if} is known as an intuitionistic fuzzy open set ($\mathcal{IFOS}$) in $\mathbb{U}$. The complement ($\mathfrak{M}_{if}{}^C$) of an $\mathcal{IFOS}$ $\mathfrak{M}_{if}$ in an $\mathcal{IFTS}(\mathbb{U}, \tau_{if})$ is called an intuitionistic fuzzy closed set($\mathcal{IFCS}$ in short) in $\mathbb{U}$. In this paper, $\mathcal{IF}$ interior is denoted by int_{if} and $\mathcal{IF}$ closure is denoted by cl_{if}.

Definition 2.4. [3] Let $(\mathbb{U}, \tau_{if})$ be an $\mathcal{IFTS}$ and $\mathfrak{M}_{if} = \{\langle \mathbb{u}, \mu_{\mathfrak{M}_{if}}(\mathbb{u}), \upsilon_{\mathfrak{M}_{if}}(\mathbb{u})\rangle : \mathbb{u} \in \mathbb{U}\}$ be an $\mathcal{IFS}$ in $\mathbb{U}$. Then the interior and closure of the above $\mathcal{IFS}$ are defined as follows:

(i) $int_{if}(\mathfrak{M}_{if}) = \cup \{\mathcal{G}_{if} \mid \mathcal{G}_{if}$ is an $\mathcal{IFOS}$ in $\mathbb{U}$ and $\mathcal{G}_{if} \subseteq \mathfrak{M}_{if}\}$

(ii) $cl_{if}(\mathfrak{M}_{if}) = \cap \{\mathcal{K}_{if} \mid \mathcal{K}_{if}$ is an $\mathcal{IFCS}$ in $\mathbb{U}$ and $\mathfrak{M}_{if} \subseteq \mathcal{K}_{if}\}$

Definition 2.5. [15] $\mathcal{IF}w$-closed set ($\mathcal{IF}w\mathcal{CS}$ in short) or $\mathcal{IF}\hat{g}$-closed ($\mathcal{IF}\,\hat{g}\mathcal{CS}$ in short) if $cl_{if}(\mathcal{A}_{if}) \subseteq \mathcal{O}$ whenever $\mathcal{A}_{if} \subseteq \mathcal{O}_{if}$ and $\mathcal{O}_{if}$ is $\mathcal{IF}s\mathcal{O}$.

Definition 2.6. [10] An $\mathcal{IFS}$ $\mathfrak{M}_{if}$ of an $\mathcal{IFTS}$ $(\mathbb{U}, \tau_{if})$ is called an $\mathcal{IF}\hat{g}^{*}s\mathcal{C}$, if $scl_{if}(\mathfrak{M}_{if}) \subseteq \mathcal{O}_{if}$ whenever $\mathfrak{M}_{if} \subseteq \mathcal{O}_{if}$ and $\mathcal{O}_{if}$ is any $\mathcal{IF}\hat{g}\mathcal{O}$ in $(\mathbb{U}, \tau_{if})$.

Definition 2.7. [8] Let $f:(\mathbb{U},\tau_{\mathfrak{if}}) \to (\mathbb{V},\sigma_{\mathfrak{if}})$ be a mapping. Then f is said to be $\mathcal{IFCM}$ if $f(\mathcal{N}_{\mathfrak{if}})$ is $\mathcal{IFCS}$ in $(\mathbb{V},\sigma_{\mathfrak{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathfrak{if}}$ in $(\mathbb{U},\tau_{\mathfrak{if}})$.

Definition 2.8. Let $f:(\mathbb{U},\tau_{\mathfrak{if}}) \to (\mathbb{V},\sigma_{\mathfrak{if}})$ be a mapping. Then f is said to be

(i) An $\mathcal{IF}\mathscr{s}\mathcal{CM}$ [11] if $f(\mathcal{N}_{\mathfrak{if}})$ is $\mathcal{IF}\mathscr{s}\mathcal{C}$ in $(\mathbb{V},\sigma_{\mathfrak{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathfrak{if}}$ in$(\mathbb{U},\tau_{\mathfrak{if}})$.

(ii) An $\mathcal{IF}\alpha\mathcal{CM}$ [12] if $f(\mathcal{N}_{\mathfrak{if}})$ is $\mathcal{IF}\alpha\mathcal{C}$ in$(\mathbb{V}, \sigma_{\mathfrak{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathfrak{if}}$ in$(\mathbb{U},\tau_{\mathfrak{if}})$.

(iii) An $\mathcal{IF}\mathscr{g}^*\mathcal{CM}$ [9] if $f(\mathcal{N}_{\mathfrak{if}})$ is $\mathcal{IF}\mathscr{g}^*\mathcal{C}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathfrak{if}}$ in $(\mathbb{U},\tau_{\mathfrak{if}})$.

(iv) An $\mathcal{IF}\mathscr{w}\mathcal{CM}$ [12] or $\mathcal{IF}$ $\widehat{\mathscr{g}}\mathcal{CM}$ if $f(\mathcal{N}_{\mathfrak{if}})$ is $\mathcal{IF}\widehat{\mathscr{g}}\mathcal{C}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathfrak{if}}$ in $(\mathbb{U},\tau_{\mathfrak{if}})$.

(v) An $\mathcal{IF}\mathscr{g}^*\mathscr{s}\mathcal{CM}$ [9] if $f(\mathcal{N}_{\mathfrak{if}})$ is $\mathcal{IF}\mathscr{g}^*\mathscr{s}\mathcal{C}$ in $(\mathbb{V},\sigma_{\mathfrak{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathfrak{if}}$ in $(\mathbb{U},\tau_{\mathfrak{if}})$.

(vi) An $\mathcal{IF}\Psi\mathcal{CM}$ [8] if $f(\mathcal{N}_{\mathfrak{if}})$ is $\mathcal{IF}\Psi\mathcal{C}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$ for every $\mathcal{IFCS}$ $\mathcal{N}_{\mathfrak{if}}$ in $(\mathbb{U},\tau_{\mathfrak{if}})$.

III. INTUITIONISTIC FUZZY $\widehat{\mathscr{g}}^*$ SEMI OPEN MAPPING IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

Definition 3.1. Let $f:(\mathbb{U},\tau_{\mathfrak{if}}) \to (\mathbb{V},\sigma_{\mathfrak{if}})$ be a mapping. Then f is said to be $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$ if $f(\mathfrak{M}_{\mathfrak{if}})$ is $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{O}$ in $(\mathbb{V},\sigma_{\mathfrak{if}})$ for every $\mathcal{IFOS}$ $\mathfrak{M}_{\mathfrak{if}}$ in $(\mathbb{U},\tau_{\mathfrak{if}})$.

Example 3.2. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}\}$, $\mathbb{V} = \{\mathfrak{g}, \mathfrak{h}\}$, $\tau_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{M}_{\mathfrak{if}}, \tilde{1}\}$ and $\sigma_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{N}_{\mathfrak{if}}, \tilde{1}\}$ where $\mathfrak{M}_{\mathfrak{if}}$=={$<\mathfrak{e}$, 0.4, 0.6$>$, $<\mathfrak{f}$, 0.42, 0.58$>$} and $\mathfrak{N}_{\mathfrak{if}}$={$<\mathfrak{g}$, 0.45, 0.55$>$, $<\mathfrak{h}$, 0.46, 0.54$>$}. Then $(\mathbb{U},\tau_{\mathfrak{if}})$ and $(\mathbb{V},\sigma_{\mathfrak{if}})$ $\mathcal{IFTS}$s. We define a mapping $f:(\mathbb{U},\tau_{\mathfrak{if}}) \to (\mathbb{V},\sigma_{\mathfrak{if}})$ by $f(\mathfrak{e}) = \mathfrak{g}$ and $f(\mathfrak{f}) = \mathfrak{h}$. Then f is $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.

Theorem 3.3. The following statements are true.

a) Every $\mathcal{IFOM}$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.
b) Every $\mathcal{IF}\mathscr{s}\mathcal{OM}$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.
c) Every $\mathcal{IF}\alpha\mathcal{OM}$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.
d) Every $\mathcal{IF}\Psi\mathcal{OM}$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.
e) Every $\mathcal{IF}\mathscr{g}^*\mathcal{OM}$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.
f) Every $\mathcal{IF}\mathscr{g}^*\mathscr{s}\mathcal{OM}$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.

Proof:

(a) Let $f:(\mathbb{U},\tau_{\mathfrak{if}}) \to (\mathbb{V},\sigma_{\mathfrak{if}})$ be an $\mathcal{IFCM}$. Let $\mathfrak{N}_{\mathfrak{if}}$ be an $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{\mathfrak{if}})$. Since f is an $\mathcal{IFOM}$, $f(\mathfrak{N}_{\mathfrak{if}})$ is an $\mathcal{IFO}$ set in$(\mathbb{V},\sigma_{\mathfrak{if}})$. Since every $\mathcal{IFOS}$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$, $f(\mathfrak{N}_{\mathfrak{if}})$ is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ in$(\mathbb{V},\sigma_{\mathfrak{if}})$. Hence f is an $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.

The proofs for (b), (c), (d), (e) and (f) are similar because every $\mathcal{IF}\mathscr{s}\mathcal{O}$, $\mathcal{IF}\alpha\mathcal{O}$, $\mathcal{IF}\Psi\mathcal{O}$, $\mathcal{IF}\mathscr{g}^*\mathcal{O}$, and $\mathcal{IF}\mathscr{g}^*\mathscr{s}\mathcal{OS}$s are $\mathcal{IF}\widehat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$.

Remark 3.4. The converse of the statements in the above theorem is not true. The examples below confirm them clearly.

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Example 3.5. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}, \mathfrak{g}\}$, $\mathbb{V} = \{\mathfrak{h}, \mathfrak{i}, \mathfrak{j}\}$, $\tau_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{M}_{\mathfrak{if}}, \tilde{1}\}$ and $\sigma_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{N}_{\mathfrak{if}}, \tilde{1}\}$ where $\mathfrak{M}_{\mathfrak{if}}=\{<\mathfrak{e}, 0.4, 0.6>, <\mathfrak{f}, 0.42, 0.58>, <\mathfrak{g}, 0.41, 0.59>\}$ and $\mathfrak{N}_{\mathfrak{if}}=\{<\mathfrak{h}, 0.45, 0.55>, <\mathfrak{i}, 0.46, 0.54>, <\mathfrak{j}, 0.47, 0.53>\}$. Then $(\mathbb{U}, \tau_{\mathfrak{if}})$ and $(\mathbb{V}, \sigma_{\mathfrak{if}})$ are $\mathcal{IFTS}$s. We define a mapping $f: (\mathbb{U}, \tau_{\mathfrak{if}}) \to (\mathbb{V}, \sigma_{\mathfrak{if}})$ by $f(\mathfrak{e}) = \mathfrak{h}$, $f(\mathfrak{f}) = \mathfrak{i}$ and $f(\mathfrak{g}) = \mathfrak{j}$. Here, f is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OM}$ but not an $\mathcal{IFOM}$, because $\mathcal{IFS}$ $\mathfrak{M}_{\mathfrak{if}} = \{<\mathfrak{e}, 0.4, 0.6>, <\mathfrak{f}, 0.42, 0.58>, <\mathfrak{g}, 0.41, 0.59>\}$ is an $\mathcal{IFOS}$ in $(\mathbb{U}, \tau_{\mathfrak{if}})$ but $f(\mathfrak{M}_{\mathfrak{if}})$ is not an $\mathcal{IFOS}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$.

Example 3.6. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}, \mathfrak{g}\}$, $\mathbb{V} = \{\mathfrak{h}, \mathfrak{i}, \mathfrak{j}\}$, $\tau_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{M}_{\mathfrak{if}}, \tilde{1}\}$ and $\sigma_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{N}_{\mathfrak{if}}, \tilde{1}\}$ where $\mathfrak{M}_{\mathfrak{if}}=\{<\mathfrak{e}, 0.22, 0.72>, <\mathfrak{f}, 0.25, 0.68>, <\mathfrak{g}, 0.3, 0.69>\}$ and $\mathfrak{N}_{\mathfrak{if}}=\{<\mathfrak{h}, 0.4, 0.6>, <\mathfrak{i}, 0.46, 0.54>, <\mathfrak{j}, 0.47, 0.53>\}$. Then $(\mathbb{U}, \tau_{\mathfrak{if}})$ and $(\mathbb{V}, \sigma_{\mathfrak{if}})$ are $\mathcal{IFTS}$s. We define a mapping $f: (\mathbb{U}, \tau_{\mathfrak{if}}) \to (\mathbb{V}, \sigma_{\mathfrak{if}})$ by $f(\mathfrak{e}) = \mathfrak{h}$, $f(\mathfrak{f}) = \mathfrak{i}$ and $f(\mathfrak{g}) = \mathfrak{j}$. Then f is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OM}$ but not an $\mathcal{IF}s\mathcal{OM}$, since $\mathcal{IFS}$ $\mathfrak{M}_{\mathfrak{if}} = \{<\mathfrak{e}, 0.22, 0.72>, <\mathfrak{f}, 0.25, 0.68>, <\mathfrak{g}, 0.3, 0.69>\}$ is an $\mathcal{IFO}$ set in $(\mathbb{U}, \tau_{\mathfrak{if}})$ but $f(\mathfrak{M}_{\mathfrak{if}})$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OS}$ and not an $\mathcal{IF}s\mathcal{OS}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$.

Example 3.7. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}, \mathfrak{g}\}$, $\mathbb{V} = \{\mathfrak{h}, \mathfrak{i}, \mathfrak{j}\}$, $\tau_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{M}_{\mathfrak{if}}, \tilde{1}\}$ and $\sigma_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{N}_{\mathfrak{if}}, \tilde{1}\}$ where $\mathfrak{M}_{\mathfrak{if}}=\{<\mathfrak{e}, 0.5, 0.48>, <\mathfrak{f}, 0.6, 0.4>, <\mathfrak{g}, 0.478\ \ 0.52>\}$ and $\mathfrak{N}_{\mathfrak{if}}=\{<\mathfrak{h}, 0.46, 0.52>, <\mathfrak{i}, 0.34, 0.65>, <\mathfrak{j}, 0.42, 0.58>\}$. Then $(\mathbb{U}, \tau_{\mathfrak{if}})$ and $(\mathbb{V}, \sigma_{\mathfrak{if}})$ are $\mathcal{IFTS}$s. We define a mapping $f: (\mathbb{U}, \tau_{\mathfrak{if}}) \to (\mathbb{V}, \sigma_{\mathfrak{if}})$ by $f(\mathfrak{e}) = \mathfrak{h}$, $f(\mathfrak{f}) = \mathfrak{i}$ and $f(\mathfrak{g}) = \mathfrak{j}$. Then f is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OM}$ but not an $\mathcal{IF}\alpha\mathcal{OM}$, since $\mathcal{IFS}$ $\mathfrak{M}_{\mathfrak{if}} = \{<\mathfrak{e}, 0.5, 0.48>, <\mathfrak{f}, 0.6, 0.4>, <\mathfrak{g}, 0.478\ \ 0.52>\}$ is an $\mathcal{IFOS}$ set in $(\mathbb{U}, \tau_{\mathfrak{if}})$ but $f(\mathfrak{M}_{\mathfrak{if}})$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OS}$ and not an $\mathcal{IF}\alpha\mathcal{OS}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$.

Example 3.8. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}, \mathfrak{g}\}$, $\mathbb{V} = \{\mathfrak{h}, \mathfrak{i}, \mathfrak{j}\}$, $\tau_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{M}_{\mathfrak{if}}, \tilde{1}\}$ and $\sigma_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{N}_{\mathfrak{if}}, \tilde{1}\}$ where $\mathfrak{M}_{\mathfrak{if}}=\{<\mathfrak{e}, 0.4, 0.6>, <\mathfrak{f}, 0.3, 0.7>, <\mathfrak{g}, 0.28\ \ 0.71>\}$ and $\mathfrak{N}_{\mathfrak{if}}=\{<\mathfrak{h}, 0.41, 0.59>, <\mathfrak{i}, 0.34, 0.66>, <\mathfrak{j}, 0.32, 0.68>\}$. Then $(\mathbb{U}, \tau_{\mathfrak{if}})$ and $(\mathbb{V}, \sigma_{\mathfrak{if}})$ are $\mathcal{IFTS}$s. We define a mapping $f: (\mathbb{U}, \tau_{\mathfrak{if}}) \to (\mathbb{V}, \sigma_{\mathfrak{if}})$ by $f(\mathfrak{e}) = \mathfrak{h}$, $f(\mathfrak{f}) = \mathfrak{i}$ and $f(\mathfrak{g}) = \mathfrak{j}$. Then f is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OM}$ but not an $\mathcal{IF}\Psi\mathcal{OM}$, since $\mathcal{IFS}$ $\mathfrak{M}_{\mathfrak{if}} = \{<\mathfrak{e}, 0.4, 0.6>, <\mathfrak{f}, 0.3, 0.7>, <\mathfrak{g}, 0.28\ \ 0.71>\}$ is an $\mathcal{IFOS}$ in $(\mathbb{U}, \tau_{\mathfrak{if}})$ but $f(\mathfrak{M}_{\mathfrak{if}})$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OS}$ and not an $\mathcal{IF}\Psi\mathcal{OS}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$.

Example 3.9. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}, \mathfrak{g}\}$, $\mathbb{V} = \{\mathfrak{h}, \mathfrak{i}, \mathfrak{j}\}$, $\tau_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{M}_{\mathfrak{if}}, \tilde{1}\}$ and $\sigma_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{N}_{\mathfrak{if}}, \tilde{1}\}$ where $\mathfrak{M}_{\mathfrak{if}}=\{<\mathfrak{e}, 0.45, 0.55>, <\mathfrak{f}, 0.6, 0.4>, <\mathfrak{g}, 0.5\ \ 0.4>\}$ and $\mathfrak{N}_{\mathfrak{if}}=\{<\mathfrak{h}, 0.4, 0.6>, <\mathfrak{i}, 0.3, 0.7>, <\mathfrak{j}, 0.2, 0.8>\}$. Then $(\mathbb{U}, \tau_{\mathfrak{if}})$ and $(\mathbb{V}, \sigma_{\mathfrak{if}})$ are $\mathcal{IFTS}$s. We define a mapping $f: (\mathbb{U}, \tau_{\mathfrak{if}}) \to (\mathbb{V}, \sigma_{\mathfrak{if}})$ by $f(\mathfrak{e}) = \mathfrak{h}$, $f(\mathfrak{f}) = \mathfrak{i}$ and $f(\mathfrak{g}) = \mathfrak{j}$. Then f is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OM}$ but not an $\mathcal{IF}g^{*}\mathcal{OM}$, since $\mathcal{IFS}$ $\mathfrak{M}_{\mathfrak{if}} ==\{<\mathfrak{e}, 0.45, 0.55>, <\mathfrak{f}, 0.6, 0.4>, <\mathfrak{g}, 0.5\ \ 0.4>\}$ is an $\mathcal{IFOS}$ in $(\mathbb{U}, \tau_{\mathfrak{if}})$ but $f(\mathfrak{M}_{\mathfrak{if}})$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OS}$ and not an $\mathcal{IF}g^{*}\mathcal{OS}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$.

Example 3.10. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}, \mathfrak{g}\}$, $\mathbb{V} = \{\mathfrak{h}, \mathfrak{i}, \mathfrak{j}\}$, $\tau_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{M}_{\mathfrak{if}}, \tilde{1}\}$ and $\sigma_{\mathfrak{if}} = \{\tilde{0}, \mathfrak{N}_{\mathfrak{if}}, \tilde{1}\}$ where $\mathfrak{M}_{\mathfrak{if}}=\{<\mathfrak{e}, 0.55, 0.20>, <\mathfrak{f}, 0.6, 0.4>, <\mathfrak{g}, 0.72\ \ 0.28>\}$ and $\mathfrak{N}_{\mathfrak{if}}=\{<\mathfrak{h}, 0.25, 0.75>, <\mathfrak{i}, 0.3, 0.7>, <\mathfrak{j}, 0.27, 0.73>\}$. Then $(\mathbb{U}, \tau_{\mathfrak{if}})$ and $(\mathbb{V}, \sigma_{\mathfrak{if}})$ are $\mathcal{IFTS}$s. We define a mapping $f(\mathbb{U}, \tau_{\mathfrak{if}}) \to (\mathbb{V}, \sigma_{\mathfrak{if}})$ by $f(\mathfrak{e}) = \mathfrak{h}$, $f(\mathfrak{f}) = \mathfrak{i}$ and $f(\mathfrak{g}) = \mathfrak{j}$. Then f is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OM}$ but not an $\mathcal{IF}g^{*}s\mathcal{OM}$, since $\mathcal{IFS}$ $\mathfrak{M}_{\mathfrak{if}}=\{<\mathfrak{e}, 0.55, 0.20>, <\mathfrak{f}, 0.6, 0.4>, <\mathfrak{g}, 0.72\ \ 0.28>\}$ is an $\mathcal{IFOS}$ in $(\mathbb{U}, \tau_{\mathfrak{if}})$ but $f(\mathfrak{M}_{\mathfrak{if}})$ is an $\mathcal{IF}\widehat{g}^{*}s\mathcal{OS}$ and not an $\mathcal{IF}g^{*}s\mathcal{OS}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$.

Remark. 3.11: $\mathcal{IF}\widehat{g}^*s\mathcal{OM}$ and $\mathcal{IF}\widehat{g}\mathcal{OM}$ are independent. The examples given below testify it.

Example 3.12. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}, \mathfrak{g}\}$, $\mathbb{V} = \{\mathfrak{h}, \mathfrak{i}, \mathfrak{j}\}$, $\tau_{\mathfrak{if}} = \{\tilde{0},\ \mathfrak{M}_{\mathfrak{if}},\ \tilde{1}\}$ and $\sigma_{\mathfrak{if}} = \{\tilde{0},\ \mathfrak{N}_{\mathfrak{if}},\ \tilde{1}\}$ where $\mathfrak{M}_{\mathfrak{if}}=\{<\mathfrak{e},\ 0.3,\ 0.7>,\ <\mathfrak{f},\ 0.4,\ 0.6>,\ <\mathfrak{g},\ 0.25\ \ 0.75>\}$ and $\mathfrak{N}_{\mathfrak{if}}=\{<\mathfrak{h},\ 0.2,\ 0.78>,\ <\mathfrak{i},\ 0.3,\ 0.7>,\ <\mathfrak{j},\ 0.22,\ 0.78>\}$. Then $(\mathbb{U}, \tau_{\mathfrak{if}})$ and $(\mathbb{V}, \sigma_{\mathfrak{if}})$ are $\mathcal{IFTS}$s. We define a mapping $f: (\mathbb{U}, \tau_{\mathfrak{if}}) \to (\mathbb{V}, \sigma_{\mathfrak{if}})$ by $f(\mathfrak{e}) = \mathfrak{h}$, $f(\mathfrak{f}) = \mathfrak{i}$ and $f(\mathfrak{g}) = \mathfrak{j}$. Then $f(\mathfrak{M}_{\mathfrak{if}})$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{O}$ set in $(\mathbb{V}, \sigma_{\mathfrak{if}})$ and not an $\mathcal{IF}\widehat{g}\mathcal{O}$ set in $(\mathbb{V}, \sigma_{\mathfrak{if}})$ where $\mathfrak{M}_{\mathfrak{if}} = \{<\mathfrak{e},\ 0.3,\ 0.7>, <\mathfrak{f},\ 0.4,\ 0.6>, <\mathfrak{g},\ 0.25\ \ 0.75>\}$ is an $\mathcal{IFOS}$ set in $(\mathbb{U}, \tau_{\mathfrak{if}})$. Therefore f is an $\mathcal{IF}\widehat{g}^*s\mathcal{OM}$ but not an $\mathcal{IF}\widehat{g}\mathcal{OM}$.

Example 3.13. Let $\mathbb{U} = \{\mathfrak{e}, \mathfrak{f}, \mathfrak{g}\}$, $\mathbb{V} = \{\mathfrak{h}, \mathfrak{i}, \mathfrak{j}\}$, $\tau_{\mathfrak{if}} = \{\tilde{0},\ \mathfrak{M}_{\mathfrak{if}},\ \tilde{1}\}$ and $\sigma_{\mathfrak{if}} = \{\tilde{0},\ \mathfrak{N}_{\mathfrak{if}},\ \tilde{1}\}$ where $\mathfrak{M}_{\mathfrak{if}}=\{<\mathfrak{e},\ 0.22,\ 0.78>,\ <\mathfrak{f},\ 0.7,\ 0.3>,\ <\mathfrak{g},\ 0.35\ \ 0.65>\}$ and $\mathfrak{N}_{\mathfrak{if}}=\{<\mathfrak{h},\ 0.3,\ 0.7>,\ <\mathfrak{i},\ 0.4,\ 0.6>,\ <\mathfrak{j},\ 0.37,\ 0.6>\}$. Then $(\mathbb{U}, \tau_{\mathfrak{if}})$ and $(\mathbb{V}, \sigma_{\mathfrak{if}})$ are $\mathcal{IFTS}$s. We define a mapping $f(\mathbb{U}, \tau_{\mathfrak{if}}) \to (\mathbb{V}, \sigma_{\mathfrak{if}})$ by $f(\mathfrak{e}) = \mathfrak{h}$, $f(\mathfrak{f}) = \mathfrak{i}$ and $f(\mathfrak{g}) = \mathfrak{j}$. Then $f(\mathfrak{M}_{\mathfrak{if}})$ is an $\mathcal{IF}\widehat{g}\mathcal{OS}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$ and not an $\mathcal{IF}\widehat{g}^*s\mathcal{OS}$ in $(\mathbb{V}, \sigma_{\mathfrak{if}})$ where $\mathfrak{M}_{\mathfrak{if}} = \{<\mathfrak{e},\ 0.22,\ 0.78>, <\mathfrak{f},\ 0.7,\ 0.3>, <\mathfrak{g},\ 0.35\ \ 0.65>\}$ is an $\mathcal{IFOS}$ in $(\mathbb{U}, \tau_{\mathfrak{if}})$. Therefore f is an $\mathcal{IF}\widehat{g}\mathcal{OM}$ but not an $\mathcal{IF}\widehat{g}^*s\mathcal{OM}$.

The diagram below depicts the interrelationship of $\mathcal{IF}\widehat{g}^*s\mathcal{OM}$ with some of the other $\mathcal{IF}$ mappings.

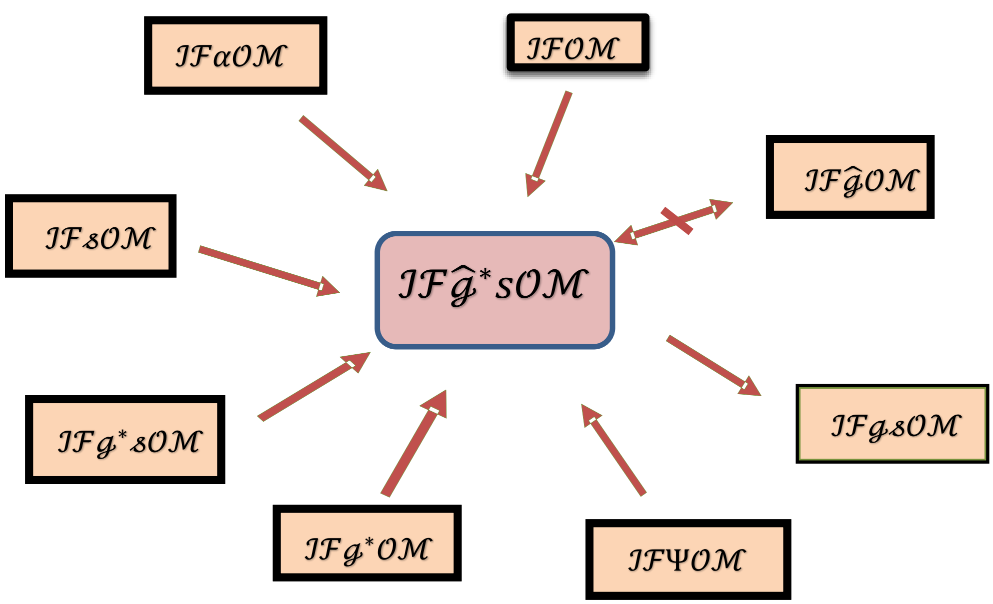

Figure 3.1

Theorem 3.14. If $f: (\mathbb{U}, \tau_{\mathfrak{if}}) \to (\mathbb{V}, \sigma_{\mathfrak{if}})$ is an $\mathcal{IF}\widehat{g}^*s\mathcal{OM}$ then $int_{\mathfrak{if}}\,(f^{-1}(\mathfrak{M}_{\mathfrak{if}})) \subseteq f^{-1}(\widehat{g}^*sint_{\mathfrak{if}}\,(\mathfrak{M}_{\mathfrak{if}}))$ for every $\mathcal{IFS}$ $\mathfrak{M}_{\mathfrak{if}}$ of $(\mathbb{V}, \sigma_{\mathfrak{if}})$.

Proof: Let $\mathfrak{M}_{\text{if}}$ be an $\mathcal{IFS}$ of $(\mathbb{V},\sigma_{\text{if}})$. Then $int_{\text{if}}\,(f^{-1}(\mathfrak{M}_{\text{if}}))$ is an $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{\text{if}})$. Since f is an $\mathcal{IF}\hat{g}^*s\mathcal{OM}$, $f(int_{\text{if}}\,(f^{-1}(\mathfrak{M}_{\text{if}})))$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{V},\sigma_{\text{if}})$. And hence $f(int_{\text{if}}\,(f^{-1}(\mathfrak{M}_{\text{if}}))) \subseteq \hat{g}^*sint_{\text{if}}(f(f^{-1}(\mathfrak{M}_{\text{if}}))) \subseteq \hat{g}^*sint_{\text{if}}(\mathfrak{M}_{\text{if}})$. Thus $int_{\text{if}}\,(f^{-1}(\mathfrak{M}_{\text{if}})) \subseteq f^{-1}(\hat{g}^*sint_{\text{if}}\,(\mathfrak{M}_{\text{if}}))$.

Theorem 3.15. A mapping $f:(\mathbb{U},\tau_{\text{if}}) \to (\mathbb{V},\sigma_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{O}$ iff for every $\mathcal{IFOS}$ $\mathfrak{M}_{\text{if}}$ of $(\mathbb{U},\tau_{\text{if}})$, $f(int_{\text{if}}\,(\mathfrak{M}_{\text{if}})) \subseteq \hat{g}^*sint_{\text{if}}\,(f(\mathfrak{M}_{\text{if}}))$.

Proof: Necessity: Let $f:(\mathbb{U},\tau_{\text{if}}) \to (\mathbb{V},\sigma_{\text{if}})$ be an $\mathcal{IF}\hat{g}^*s\mathcal{OM}$ and $\mathfrak{M}_{\text{if}}$ is an $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{\text{if}})$. Now $int_{\text{if}}\,(\mathfrak{M}_{\text{if}}) = \mathfrak{M}_{\text{if}}$ which implies that $f(int_{\text{if}}\,(\mathfrak{M}_{\text{if}})) \subseteq f(\mathfrak{M}_{\text{if}})$. Since f is an $\mathcal{IF}\hat{g}^*s\mathcal{OM}$, $f(int_{\text{if}}\,(\mathfrak{M}_{\text{if}}))$ is $\mathcal{IF}\hat{g}^*s\mathcal{O}$ in $(\mathbb{V},\sigma_{\text{if}})$ such that $f(int_{\text{if}}\,(\mathfrak{M}_{\text{if}})) \subseteq f(\mathfrak{M}_{\text{if}})$. Therefore $f(int_{\text{if}}\,(\mathfrak{M}_{\text{if}}) \subseteq \hat{g}^*sint_{\text{if}}\,(f(\mathfrak{M}_{\text{if}}))$.

Sufficiency: Suppose that $\mathfrak{M}_{\text{if}}$ is an $\mathcal{IFOS}$ of $(\mathbb{U},\tau_{\text{if}})$. Then $f(\mathfrak{M}_{\text{if}}) = f(int_{\text{if}}\,(\mathfrak{M}_{\text{if}})) \subseteq \hat{g}^*sint_{\text{if}}\,(f(\mathfrak{M}_{\text{if}}))$. But $\hat{g}^*sint_{\text{if}}\,(f(\mathfrak{M}_{\text{if}})) \subseteq f(\mathfrak{M}_{\text{if}})$. Consequently $f(\mathfrak{M}_{\text{if}}) = \hat{g}^*sint_{\text{if}}\,(f(\mathfrak{M}_{\text{if}})) \Rightarrow f(\mathfrak{M}_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ of $(\mathbb{V},\sigma_{\text{if}})$ and hence f is an $\mathcal{IF}\hat{g}^*s\mathcal{OM}$.

Theorem 3.16. A bijective mapping $f:(\mathbb{U},\tau_{\text{if}}) \to (\mathbb{V},\sigma_{\text{if}})$ is $\mathcal{IF}\hat{g}^*s\mathcal{OM}$ iff for every $\mathcal{IFS}$ $\mathfrak{N}_{\text{if}}$ of $(\mathbb{V},\sigma_{\text{if}})$ and for every $\mathcal{IFCS}$ $\mathcal{H}_{\text{if}}$ in $(\mathbb{U},\tau_{\text{if}})$ containing $f^{-1}(\mathfrak{N}_{\text{if}})$, there is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ $\mathfrak{M}_{\text{if}}$ of $(\mathbb{V},\sigma_{\text{if}})$ such that $\mathfrak{N}_{\text{if}} \subseteq \mathfrak{M}_{\text{if}}$ and $f^{-1}(\mathfrak{M}_{\text{if}}) \subseteq \mathcal{H}_{\text{if}}$.

Proof. Necessity: Let $\mathfrak{N}_{\text{if}}$ be any $\mathcal{IFS}$ in $(\mathbb{V},\sigma_{\text{if}})$. Let $\mathcal{H}_{\text{if}}$ be $\mathcal{IFCS}$ in $(\mathbb{U},\tau_{\text{if}})$ such that $f^{-1}(\mathfrak{N}_{\text{if}}) \subseteq \mathcal{H}_{\text{if}}$, then ${\mathcal{H}_{\text{if}}}^{C}$ is an $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{\text{if}})$. By given condition $f({\mathcal{H}_{\text{if}}}^{C})$ is $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{V},\sigma_{\text{if}})$. Let $\mathfrak{M}_{\text{if}} = (f({\mathcal{H}_{\text{if}}}^{C}))^{C}$, then $\mathfrak{M}_{\text{if}}$ is an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ in $(\mathbb{V},\sigma_{\text{if}})$ and $\mathfrak{N}_{\text{if}} \subseteq \mathfrak{M}_{\text{if}}$, since for a bijective mapping $(f({\mathcal{H}_{\text{if}}}^{C}))^{C} = f(\mathcal{H}_{\text{if}})$. Now $f^{-1}(\mathfrak{M}_{\text{if}}) = f^{-1}(f({\mathcal{H}_{\text{if}}}^{C}))^{C} = (f^{-1}(f({\mathcal{H}_{\text{if}}}^{C})))^{C} \subseteq \mathcal{H}_{\text{if}}$.

Sufficiency: Let $\mathfrak{M}_{\text{if}}$ be any $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{\text{if}})$, then ${\mathfrak{M}_{\text{if}}}^{C}$ is an $\mathcal{IFCS}$ in $(\mathbb{U},\tau_{\text{if}})$ and $f^{-1}(f({\mathfrak{M}_{\text{if}}}^{C})) \subseteq {\mathfrak{M}_{\text{if}}}^{C}$. By given condition there exists an $\mathcal{IF}\hat{g}^*s\mathcal{CS}$ $\mathfrak{N}_{\text{if}}$ in $(\mathbb{V},\sigma_{\text{if}})$ such that $f({\mathfrak{M}_{\text{if}}}^{C}) \subseteq \mathfrak{N}_{\text{if}}$ and $f^{-1}(\mathfrak{N}_{\text{if}}) \subseteq {\mathfrak{M}_{\text{if}}}^{C}$. Hence ${\mathfrak{N}_{\text{if}}}^{C} \subseteq f(\mathfrak{M}_{\text{if}}) \subseteq f(f^{-1}(\mathfrak{N}_{\text{if}}))^{C} \subseteq (f(f^{-1}(\mathfrak{N}_{\text{if}})))^{C} \subseteq {\mathfrak{N}_{\text{if}}}^{C}$. This implies that $f(\mathfrak{M}_{\text{if}}) = {\mathfrak{N}_{\text{if}}}^{C}$. Since ${\mathfrak{N}_{\text{if}}}^{C}$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$, $f(\mathfrak{M}_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{V},\sigma_{\text{if}})$. Hence f is an $\mathcal{IF}\hat{g}^*s\mathcal{OM}$.

Theorem 3.17. If $f:(\mathbb{U},\tau_{\text{if}}) \to (\mathbb{V},\sigma_{\text{if}})$ is an $\mathcal{IFOM}$ and $g:(\mathbb{V},\sigma_{\text{if}}) \to (\mathbb{W},\eta_{\text{if}})$ is $\mathcal{IF}\hat{g}^*s\mathcal{OM}$ then $g\circ f:(\mathbb{U},\tau_{\text{if}}) \to (\mathbb{W},\eta_{\text{if}})$ is $\mathcal{IF}\hat{g}^*s\mathcal{OM}$.

Proof: Let $\mathfrak{M}_{\text{if}}$ be an $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{\text{if}})$ Then by given condition, $f(\mathfrak{M}_{\text{if}})$ is $\mathcal{IFOS}$ in $(\mathbb{V},\sigma_{\text{if}})$.
Also given that $g:(\mathbb{V},\sigma_{\text{if}}) \to (\mathbb{W},\eta_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OM}$. This implies $g(f(\mathfrak{M}_{\text{if}})) = (g\circ f)(\mathfrak{M}_{\text{if}})$ is $\mathcal{IF}\hat{g}^*s\mathcal{OS}$ in $(\mathbb{W},\eta_{\text{if}})$. Hence $g\circ f:(\mathbb{U},\tau_{\text{if}}) \to (\mathbb{W},\eta_{\text{if}})$ is an $\mathcal{IF}\hat{g}^*s\mathcal{OM}$.

Theorem 3.18. Let $f:(\mathbb{U},\tau_{\text{if}}) \to (\mathbb{V},\sigma_{\text{if}})$ and $g:(\mathbb{V},\sigma_{\text{if}}) \to (\mathbb{W},\eta_{\text{if}})$ be two mappings and let $g\circ f:(\mathbb{U},\tau_{\text{if}}) \to (\mathbb{W},\eta_{\text{if}})$ be $\mathcal{IF}\hat{g}^*s\mathcal{OM}$. Then

(a) If g is irresolute and injective, then f is $\mathcal{IF}\hat{g}^*s\mathcal{OM}$.

(b) If f is $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}-cont.$, surjective and $(\mathbb{U},\tau_{if})$ is an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}$ space, then g is $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.

Proof: (a) Let $\mathfrak{N}_{if}$ be an $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{if})$. Then $(g\circ f)(\mathfrak{N}_{if})$ is $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{O}$ in $(\mathbb{W},\eta_{if})$ and $g^{-1}((g\circ f)(\mathfrak{N}_{if}))) = f(\mathfrak{N}_{if})$ is $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{O}$ in $(\mathbb{V},\sigma_{if})$ (since g is irresolute). Hence f is $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.

(b) Let $\mathfrak{N}_{if}$ be an $\mathcal{IFOS}$ in $(\mathbb{V},\sigma_{if})$ Then $f^{-1}(\mathfrak{N}_{if})$ is $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{O}$ in $(\mathbb{U},\tau_{if})$ and $(g\circ f)(f^{-1}(\mathfrak{N}_{if})) = g(\mathfrak{N}_{if})$, which is $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{O}$ in $(\mathbb{W},\eta_{if})$ Hence g is $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{OM}$.

Theorem 3.19. A mapping $f:(\mathbb{U},\tau_{if})\to(\mathbb{V},\sigma_{if})$ is an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{O}$ mapping if $f(\hat{\mathscr{g}}^*\mathscr{s}int_{if}(\mathfrak{M}_{if}))\subseteq \hat{\mathscr{g}}^*\mathscr{s}int_{if}(f(\mathfrak{M}_{if}))$ for every $\mathfrak{M}_{if}\in(\mathbb{U},\sigma_{if})$.

Proof: Let $\mathfrak{M}_{if}$ be an $\mathcal{IFOS}$ in $(\mathbb{U},\sigma_{if})$. Then $int_{if}(\mathfrak{M}_{if})=\mathfrak{M}_{if}$. Now $f(\mathfrak{M}_{if})=f(int_{if}(\mathfrak{M}_{if}))\subseteq f(\hat{\mathscr{g}}^*\mathscr{s}int_{if}(\mathfrak{M}_{if}))\subseteq\hat{\mathscr{g}}^*\mathscr{s}int_{if}(f(\mathfrak{M}_{if}))$, by hypothesis. But $\hat{\mathscr{g}}^*\mathscr{s}int_{if}(f(\mathfrak{M}_{if}))\subseteq f(\mathfrak{M}_{if})$. Therefore $f(\mathfrak{M}_{if})$ is an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ in $(\mathbb{V},\sigma_{if})$. Hence f is an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{O}$ mapping.

Definition 3.20. An $\mathcal{IFTS}$ $(\mathbb{U},\tau_{if})$ is called $\mathcal{IF}\hat{\mathscr{g}}^*$semi $T^*_{1/2}$ space ($\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}T^*_{1/2}$ space) if every $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ is $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{if})$.

Theorem 3.21. A mapping $f:(\mathbb{U},\tau_{if})\to(\mathbb{V},\sigma_{if})$ is an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{O}$ mapping iff $int_{if}(f^{-1}(\mathfrak{N}_{if}))\subseteq f^{-1}(int_{if}(\mathfrak{N}_{if}))$ for every $\mathfrak{N}_{if}\in(\mathbb{V},\sigma_{if})$, where $(\mathbb{V},\sigma_{if})$ is an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}$ $T^*_{1/2}$ space.

Proof: Necessary Part: Let $\mathfrak{N}_{if}\in(\mathbb{V},\sigma_{if})$. Then $f^{-1}(\mathfrak{N}_{if})\subseteq(\mathbb{U},\tau_{if})$ and $int_{if}(f^{-1}(\mathfrak{N}_{if}))$ is an $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{if})$. By Hypothesis, $f(int_{if}(f^{-1}(\mathfrak{N}_{if})))$ is an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ in $(\mathbb{V},\sigma_{if})$. Since $(\mathbb{V},\sigma_{if})$ is an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}$ $T^*_{1/2}$ space, $f(int_{if}(f^{-1}(\mathfrak{N}_{if})))$ is an $\mathcal{IFOS}$ in $(\mathbb{V},\tau_{if})$. Therefore $f(int_{if}(f^{-1}(\mathfrak{N}_{if}))=int_{if}(f(int_{if}(f^{-1}(\mathfrak{N}_{if}))))\subseteq int_{if}(f(f^{-1}(\mathfrak{N}_{if})))\subseteq int_{if}(\mathfrak{N}_{if})$. This implies $int_{if}(f^{-1}(\mathfrak{N}_{if}))\subseteq f^{-1}(f(int_{if}(f^{-1}(\mathfrak{N}_{if}))))\subseteq f^{-1}(int_{if}(\mathfrak{N}_{if}))$.

Sufficient Part: Let $\mathfrak{M}_{if}$ be an $\mathcal{IFOS}$ in $(\mathbb{U},\tau_{if})$. Therefore $int_{if}(\mathfrak{M}_{if})=\mathfrak{M}_{if}$. Then $f(\mathfrak{M}_{if})\subseteq(\mathbb{V},\tau_{if})$. By hypothesis $int_{if}(f^{-1}(f((\mathfrak{M}_{if})))\subseteq f^{-1}(int_{if}(f(\mathfrak{M}_{if})))$. That is $int_{if}(\mathfrak{M}_{if})\subseteq int_{if}(f^{-1}(f((\mathfrak{M}_{if})))\subseteq f^{-1}(int_{if}(f(\mathfrak{M}_{if})))$. Therefore $\mathfrak{M}_{if}\subseteq f^{-1}(int_{if}(f(\mathfrak{M}_{if})))$. This implies $f(\mathfrak{M}_{if})\subseteq f(f^{-1}(int_{if}(f(\mathfrak{M}_{if}))))\subseteq int_{if}(f(\mathfrak{M}_{if}))\subseteq f(\mathfrak{M}_{if})$. Hence $f(\mathfrak{M}_{if})$ is an $\mathcal{IFOS}$ in $(\mathbb{V},\tau_{if})$ and hence an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{OS}$ in $(\mathbb{V},\sigma_{if})$. Thus f is an $\mathcal{IF}\hat{\mathscr{g}}^*\mathscr{s}\mathcal{O}$ mapping.

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