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# INFLUENCE OF HEAT TRANSPORT IN AN INFINITELY VARIABLE RECTANGULAR SLAB

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**Abstract**
In the present article, the investigators take into consideration a rectangular object, particularly an infinitely varying slab, and attempt to study the temperature flow, displacement function, stresses, and deflection function under specific boundary-based limitations. The expression of temperature, stress and deflection are obtained mathematically by applying conventional integral transformation methods.

Keyword and phrases**:** Transient problems, temperature distribution, thermal deflection, rectangular body, stress function.

## 1. Introduction

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Many disciplines that deal with the design and production of structural materials by taking high heating into consideration can benefit greatly from the research of temperature, displacement, stress, and deflection in diverse bodies. Furthermore, the temperature flow provides a precise and trustworthy structural analysis of the body; as a result, the structural design of spacecraft may benefit from this kind of research.

Direct thermoelastic modelling under thermal shock was investigated by Tanigawa and Komatsubara [1], Vihak et al. [2], and Adams and Bert [3] by taking rectangular plates into consideration. At various environmental conditions and temperatures Khobragade and Wankhede [4], Khobragade, and Durge [5, 6] frame inverse thermoelastic models for various solids. The quasi-static stresses in a thin circular plate caused by transient temperature applied across the upper face throughout a circle's circumference have been examined by Roychaudhari [7]. In their discussion of the effects of stress and temperature distribution under various heating sources, Choi et al. [8] took into account objects with a rectangular shape. Thakare and Khobragade [11] investigated the steady-state thermoelastic problem to determine the temperature distribution, displacement function and thermal stresses of semi-infinite rectangular plate. Also some other researchers [12 to 15] also contributed to the development of the field.
This work, deals with study of transient thermoelastic problem to determine the temperature, displacement, stress distribution and thermal deflection of an infinitely varying slab occupying the space D: $\{[x, y, z] \in R^3 : -\infty \le x \le \infty ; -\infty \le y \le \infty , -h \le x \le h$ with the stated boundary conditions. The heat conduction equation is solved with the help of integral transform techniques. The results are obtained in the form of infinite series.

## 2. Deflection and thermal momentum

Consider a thick isotropic rectangular plate occupying the space $D$. The differential equation satisfied by the deflection $\omega(\xi, \zeta, t)$ [6] is

$$\nabla^2 M_T(\xi,\zeta,t) + (1-\upsilon)\, D \nabla^u \omega(\xi,\zeta,t) = 0 \tag{1}$$

Where $\upsilon$ is the Poisson's ratio of the material, $M_T$ denote the thermal momentum of the plate and $D$ denote the flexural rigidity,

where $\nabla^2 = \dfrac{d^2}{d\xi^2} + \dfrac{d^2}{d\zeta^2}$

And the resultant thermal momentum $M_T$ is defined as

$$M_T(\xi,\zeta,t) = \alpha E \int_o^h z.T(\xi,\zeta,z,t)\, dz \tag{2}$$

Where $\alpha$ and $E$ are the coefficient of liner expansion, Young's modulus respectively. Since the edge of the rectangular plate is fixed and clammed,

$$\frac{\partial \omega}{\partial z} = 0 \text{ at } z = -h, h \tag{3}$$

## 2. Heat transfer equation

The temperature of the plate at time t satisfying the differential equation

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial \zeta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{4}$$

Where $k$ is the thermal diffusivity to the material of the plate,
Subject to the initial and boundary conditions:

$$T(\xi, \zeta, z, o) = T_0(\xi, \zeta, z) \tag{5}$$

$$\left[\frac{\partial T(\xi, \zeta, z, t)}{\partial \xi}\right]_{\xi=-\infty} = 0 \tag{6}$$

$$\left[\frac{\partial T(\xi, \zeta, z, t)}{\partial \xi}\right]_{\xi=\infty} = 0 \tag{7}$$

$$\left[\frac{\partial T(\xi, \zeta, z, t)}{\partial \zeta}\right]_{\zeta=-\infty} = 0 \tag{8}$$

$$\left[\frac{\partial T(\xi, \zeta, z, t)}{\partial \zeta}\right]_{\zeta=\infty} = 0 \tag{9}$$

$$\left[T + k_1 \frac{\partial T(\xi, \zeta, z, t)}{\partial z}\right]_{z=-h} = f(\xi, \zeta, t) \tag{10}$$

$$\left[T + k_2 \frac{\partial T(\xi, \zeta, z, t)}{\partial z}\right]_{z=h} = g(\xi, \zeta, t) \tag{11}$$

Equation (1) to (11) constitute the mathematical formulation of the problem under consideration.

## 3. Evaluation of heat transfer

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Applying Fourier double cosine transform and Marchi-Fasulo transform formula to equation (4) and on utilizing their corresponding inversion formula subjected to different constraints in equations (5) to (11), we get

$$T(\xi,\zeta,z,t)=\sum_{l,m,n=0}^{\infty}\left[\frac{p_n(z)}{\lambda_n}\left\{\frac{(\overline{f}+\overline{g})}{q^2}+\left(T_0-\frac{(\overline{f}+\overline{g})}{q^2}\right)\exp\left(-q^2kt\right)\right\}\right]\exp\left(-2j\pi(ax+by)\right)$$

Where $q^2=p^2+4\pi^2(a^2+b^2)$ (12)

$$\lambda_n=\int_{-h}^{h}P_n^2(\gamma)\,d\lambda$$

$$P_n(\gamma)=Q_n\cos(a_n\gamma)-W_n\sin(a_n\gamma)$$

$$Q_n=a_n(\alpha_3+\alpha_4)\cos(a_na)+(\beta_3-\beta_4)\sin(a_na)$$

$$W_n=(\beta_3+\beta_4)\cos(a_na)+(\alpha_4-\alpha_3)\,a_n\sin(a_na)$$

Equation (12) is the desired solution of the given problem with $\beta_3=\beta_4=1$, $\alpha_3=k_1$, $\alpha_4=k_2$.

## 4. Displacement and stress function

The displacement components $u_\xi$, $u_\varsigma$ and $u_z$ in axial directions respectively are in the integral form as

$$u_\xi=\int\left[1/E\left(\frac{\partial^2u}{\partial\varsigma^2}+\frac{\partial^2u}{\partial z^2}-v\frac{\partial^2u}{\partial\xi^2}\right)+\alpha T\right]d\xi \quad (13)$$

$$u_\varsigma=\int\left[1/E\left(\frac{\partial^2u}{\partial z^2}+\frac{\partial^2u}{\partial\xi^2}-v\frac{\partial^2u}{\partial\varsigma^2}\right)+\alpha T\right]d\varsigma \quad (14)$$

$$u_z=\int\left[1/E\left(\frac{\partial^2u}{\partial\xi^2}+\frac{\partial^2u}{\partial\varsigma^2}-v\frac{\partial^2u}{\partial z^2}\right)+\alpha T\right]dz \quad (15)$$

The stress components in terms of U are given by

$$\sigma_{\xi\xi}=\frac{\partial^2U}{\partial\varsigma^2}+\frac{\partial^2U}{\partial z^2} \quad (16)$$

$$\sigma_{\varsigma\varsigma}=\frac{\partial^2u}{\partial\xi^2}+\frac{\partial^2u}{\partial z^2} \quad (17)$$

$$\sigma_{zz}=\frac{\partial^2u}{\partial\xi^2}+\frac{\partial^2u}{\partial\varsigma^2} \quad (18)$$

Where $U$ is the Airy's function which satisfy the following relation

$$\left(\frac{\partial^2}{\partial \xi^2}+\frac{\partial^2}{\partial \varsigma^2}+\frac{\partial^2}{\partial z^2}\right)^2 U=-\alpha E\left(\frac{\partial^2}{\partial \xi^2}+\frac{\partial^2}{\partial \varsigma^2}+\frac{\partial^2}{\partial z^2}\right)T \qquad (19)$$

## 5. Evaluation of Airy's function

Substituting the value of (12) in equation (19) one obtains

$$U(\xi,\zeta,z,t)=\frac{1}{q^2}\sum_{l,m,n=0}^{\infty}\left[\frac{p_n(z)}{\lambda_n}\left\{\frac{(\bar{f}+\bar{g})}{q^2}+\left(T_0-\frac{(\bar{f}+\bar{g})}{q^2}\right)\exp\left(-q^2kt\right)\right\}\right]\exp\left(-2j\pi(a\xi+b\varsigma)\right) \quad (20)$$

## 6. Stress functions

Substituting the value of (20) in equations (16), (17) and (18) one obtains

$$\sigma_{\xi\xi}=\sum_{l,m,n=0}^{\infty}\left[\frac{4\pi^2 p_n(z)}{q^2\lambda_n}\left[F(z)-b^2\right]\left\{\left(\frac{(\bar{f}+\bar{g})}{q^2}+T_0-\frac{(\bar{f}+\bar{g})}{q^2}\right)\exp\left(-q^2kt\right)\right\}\right]\times\exp\left(-2j\pi(a\xi+b\varsigma)\right) \quad (21)$$

$$\sigma_{\varsigma\varsigma}=\sum_{l,m,n=0}^{\infty}\left[\frac{4\pi^2 p_n(z)}{q^2\lambda_n}\left[F(z)-a^2\right]\left\{\left(\frac{(\bar{f}+\bar{g})}{q^2}+T_0-\frac{(\bar{f}+\bar{g})}{q^2}\right)\exp\left(-q^2kt\right)\right\}\right]\times\exp\left(-2j\pi(a\xi+b\varsigma)\right) \quad (22)$$

$$\sigma_{zz}=\sum_{l,m,n=0}^{\infty}\left[\frac{4\pi^2 p_n(z)}{q^2\lambda_n}\left[F(z)+b^2\right]\left\{\left(\frac{(\bar{f}+\bar{g})}{q^2}+T_0-\frac{(\bar{f}+\bar{g})}{q^2}\right)\exp\left(-q^2kt\right)\right\}\right]\times\exp\left(-2j\pi(a\xi+b\varsigma)\right) \quad (23)$$

## 7. Evaluation of thermal deflection

Substituting the value of temperature distribution $T(\xi,\zeta,z,t)$ from equation (12) to the equation (2) we obtain the expression for thermal momentum as

$$\omega(\xi,\varsigma,t)=\frac{K\alpha E}{8D(1-\upsilon)}\sum_{l,m,n=0}^{\infty}\left[\frac{\left(Q_n\sin(a_n\gamma)+W_n\cos(a_n\gamma)\right)}{\lambda_n\gamma}\sin\left(\frac{m\pi z}{h}\right).[(z+h)(z-h)]\right]$$
$$\times\left\{\frac{(\bar{f}+\bar{g})}{q^2}+\left(T_0-\frac{(\bar{f}+\bar{g})}{q^2}\right)\exp\left(-q^2kt\right)\right\}\exp\left(-2j\pi(a\xi+b\varsigma)\right) \quad (24)$$

## Conclusion

Using integral transform techniques, mathematical representations of temperature, displacement, stress distribution, and thermal deflection are assessed in an indefinitely variable rectangular object exposed to various constraints. The solution for the series is convergent. The acquired temperature distribution stress and deflection can be used to create practical machines or structures for use in applications in engineering.

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