SIGNAL NUMBER IN SPLITTING GRAPHS OF GRAPHS

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ABSTRACT

A set S vertices of a graph G is a signal set of G if every vertex of G lies on $u-v$ geosig for some elements u and v in S. The minimum cardinality of a signal set of G is the signal number of G and is denoted by $sn(G)$. In this paper, we explore the concept of signal number in splitting graph of star graph, Bistar graph, cycle graph, wheel graph and fan graph some properties also to be obtained.

Keywords: Signal distance, Signal set, Signal number, splitting graph.
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1 Introduction

We consider here only the finite, simple, connected graphs with vertex set V and edge set E. For any graph G, the order is n and size is m. the degree $d(v)$ of a vertex v in $V(G)$ is the number of edges incident to v. For any vertex v in G, the open neighbourhood $N(v)$ is the set of all vertices adjacent to that v and $N[v] = N(v) \cup \{v\}$ is the closed neighbourhood of v. Let $\Delta= \Delta(G)$ and $\delta = \delta(G)$ denote for the maximum and minimum degree of G, respectively.

If G be any graph, then the complement of G is denoted by $\bar{G}$. The girth of G is denoted by $c(G)$, which is the length of the shortest cycle in G. A vertex v is said to be an extreme vertex of G, if its neighbourhood $N(v)$ induces a complete subgraph of G. If G is a connected graph, then the distance denoted by $d(x,y)$ is the length of a shortest $x-y$ path in G.

On the various study of distance in graphs, we refer to [1]. In continuation, kathiresan et.al introduced a distance parameter known as signal distance of graphs [4]. The signal distance $d_{SD}(u,v)$ between the pairs u and v is defined by

$$d_{SD}(u,v) = \min \{d(u,v) + \sum_{w \in V(G)} (degw - 2) + (degu - 1) + \deg(v-1)\}$$

Where S is the path connecting u and v, $d(u,v)$ be the length of path S and the sum $\sum_{w \in V(G)}$ runs over all the internal vertices between u and v in the path S.

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The $u - v$ signal path of length $d_{SD}(u, v)$ is also called geosig. A vertex v is known as lie on a geosig P if v is an internal vertex of P.

In [6], author introduce the notation that $L[x, y]$ consists of x and y and all vertices lying on some $x - y$ geosig of G and for a non-empty set $S \subseteq V(G)$,

$$L[S] = \bigcup_{x, y \in S} L[x, y].$$

A set $S \subseteq V(G)$ is said to be a signal set of G if $L[S] = V(G)$. The minimum cardinality of a signal set is known as signal number and is denoted by $s(G)$ [2]. A set S as a subset of $V(G)$ is known as geodetic set if $I[S] = V(G)$. The minimum cardinality of a geodetic set of G is known as geodetic number and is denoted by gG). The undefined notation and symbols we refer [2,5].

A star graph is complete bipartite graph $K_{1,n-1}$ of order n.

A bistar graph $B(m, n)$ is obtained from K_2 by attaching m edges in one vertex and n −edges in the other vertex.

The following Theorems is very much useful for the following sections.

Theorem 1.1 [6] Each extreme vertex of G belongs to every signal set of G.

Theorem 1.2[6] $sn(G) = 2$ if and only if there exist vertices u, v such that v is an u −signal vertex of G.

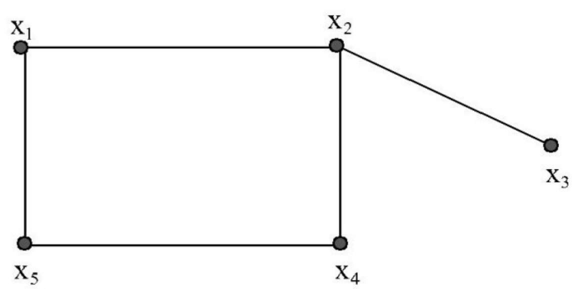
2. Signal number in splitting graph.

Definition 2.1 Let G be any connected graph of $n \geq 2$ vertices. The splitting graph $S'(G)$ of G is obtained by adding a new vertex x' of G corresponding to every vertex x of G such that x' is adjacent to every vertex of x in G.

If n is the number of vertices of G, then $2n$ is the number of vertices of $S'(G)$. We call the vertices $x_1, x_2, \dots, x_n$ are duplicated by $x'_1, x'_2, \dots, x'_n$.

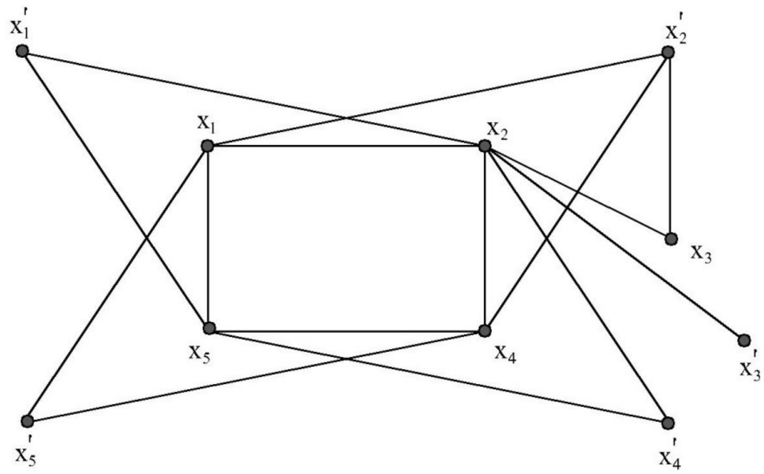
Now we find the signal of $S'(G)$ for the following graph G.

Example 2.2 Consider the graph G in Figure 2.1



G

Figure 2.1

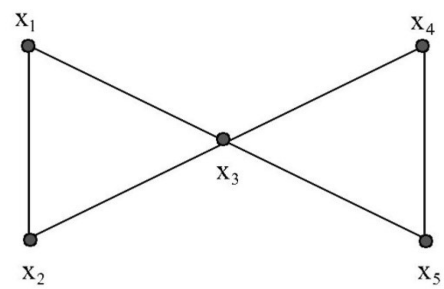


$S'(G)$

Here $S = \{x_1', x_3', x_4', x_5', x_3\}$ is a minimum signal set of $S'(G)$ and hence $sn\big(S'(G)\big) = 5$.

Remark 2.3 Every extreme of G need not be a member of every signal set of $S'(G)$.

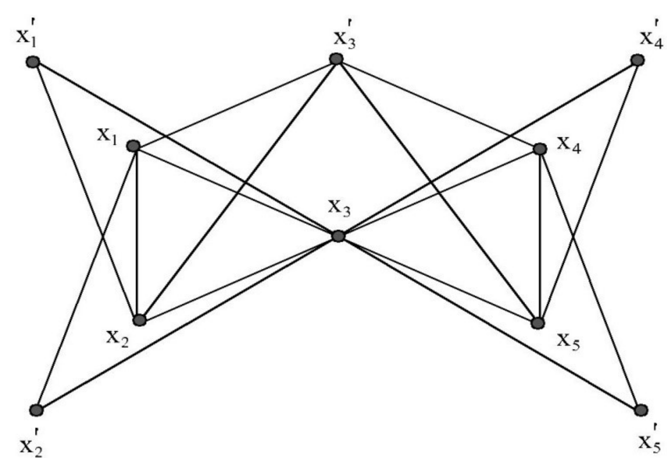
For example, Consider the graph G given in Figure 2.2.



G

Figure 2.2

Here $S' = \{x_1, x_2, x_4, x_5\}$ are the extreme vertices of G.



$S'(G)$

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It is easily verified that $S = \{x_1', x_2', x_3', x_4', x_5'\}$ is the unique minimum signal set of $S'(G)$ and so $sn(S'(G)) = 5$. But $S' \nsubseteq S$.

Theorem 2.4 Each duplicate vertex of an extreme vertex in G belong to every signal set of $S'(G)$.

Proof.

Let u be an extreme vertex of G.

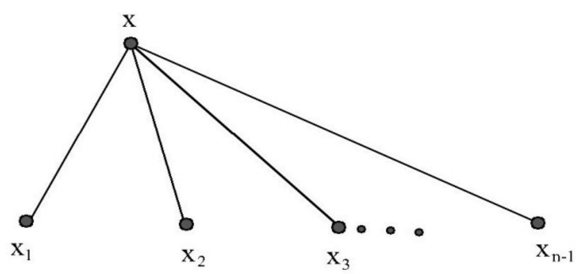
Then by definition that the subgraph induced by u is complete. That is $N[u]$ is complete.

Let S be a signal set of $S'(G)$. Therefore u' is adjacent to every vertices of the neighbours of u in G. This follows that neighbours of u' must be complete. Thus u' be the extreme vertex of $S'(G)$ and hence by Theorem1.1,$u' \in S$.

Theorem 2.5 For any integer $n \geq 3$, $sn\left(S'(K_{1,n-1})\right) = n$.

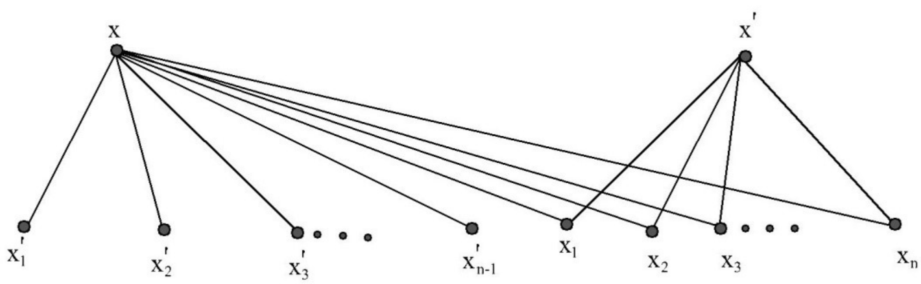
Proof.

Consider the graph $K_{1,n-1}$. Let x be the centre vertex and $x_1, x_2, \dots, x_{n-1}$ be the end vertices adjacent to x. Let $x_1', x_2', \dots, x_n'$ be the corresponding duplicated vertices of $x_1, x_2, \dots, x_n$ respectively to form $S'(K_{1,n-1})$ and is given in Figure 2.3(a).



$K_{1,n-1}$

Figure 2.3



$S'(K_{1,n-1})$

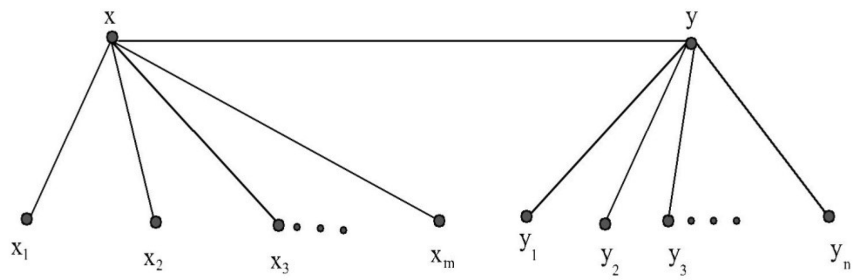
Figure 2.3(a)

Clearly $V\left(S'(K_{1,n-1})\right) = \{x, x_1, x_2, \dots, x_{n-1}, x', x_1', x_2', \dots, x_{n-1}'\}$ and $\left|V\left(S'(K_{1,n-1})\right)\right| = 2n$. Let $S = \{x_1', x_2', \dots, x_{n-1}'\}$. It is clear that S is the set of all extreme vertices of $S'(K_{1,n-1})$, and so by Theorem 1.1, $sn\left(S'(K_{1,n-1})\right) \geq |S| = n-1$. Now it can be easily verified of $S'(K_{1,n-1})$ and hence $sn\left(S'(K_{1,n-1})\right) = n$.

Theorem 2.6 For any integer $m, n \geq 2$, $sn\left(S'(B_{m,n})\right) = m+n+2$.

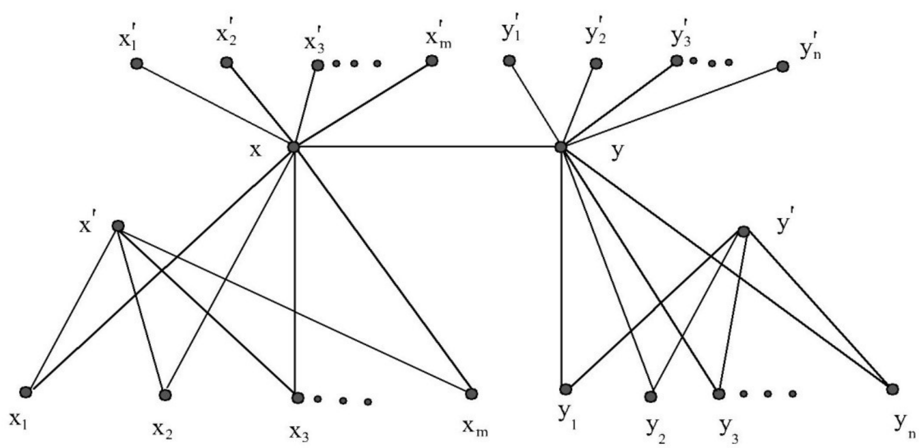
Proof.

Let x and y be central vertices of the graph $B_{m,n}$. Let $x_1, x_2, \dots, x_m$ and $y_1, y_2, \dots, y_n$ be the vertices that are adjacent with x and y respectively. Let $x', y', x'_1, x'_2, \dots, x'_m$, $y'_1, y'_2, \dots, y'_n$ be the corresponding duplicate vertices which are added thus to obtain the graph $S'(B_{m,n})$ and is given in Figure 2.4(a).



$B_{m,n}$

Figure 2.4



$S'(B_{m,n})$

Figure 2.4(a)

Clearly $V\left(S'(B_{m,n})\right) = \{x, x_1, x_2, \dots, x_m, y_1, y_2, \dots, y_n, x'_1, x'_2, \dots, x'_m, y'_1, y'_2, \dots, y'_n,$ $x', y'\}$ and $\left|V\left(S'(B_{m,n})\right)\right| = 2m+2n+4$. Let $S = \{x'_1, x'_2, \dots, x'_m, y'_1, y'_2, \dots, y'_n\}$ be the set of all extreme vertices of $S'(B_{m,n})$ and so by Theorem 1.1, $sn\left(S'(B_{m,n})\right) \geq |S| = m+n$. Let S_1 be a minimum signal set of $S'(B_{m,n})$. Clearly $S \subseteq S_1$. Now, there is a geosig path between x' and y' which containing the remaining vertices $x_1, x_2, \dots, x_m, y_1, y_2, \dots, y_n$ which are not in S. Also if

$x', y' \in S_1$ then $S_1 = S \cup \{x', y'\}$ becomes a signal set of minimum set of minimum cardinality $m + n + 2$. Hence $sn\left(S'\left(B_{m,n}\right)\right) = m + n + 2$

Theorem 2.7 For any integer $n \geq 2$, $sn\big(S'(C_n)\big) = n$.

Proof.

Consider the cycle graph with vertex set $\{x_1, x_2, \dots, x_n\}$. Let $x_1', x_2', \dots, x_n'$ be the duplicated vertex corresponding to $x_1, x_2, \dots, x_n$ which are added to C_n and obtained the graph $S'(C_n)$.

Clearly $V\big(S'(C_n)\big) = \{x_1, x_2, \dots, x_n, x_1', x_2', \dots, x_n'\}$ and thus $\left|V\big(S'(C_n)\big)\right| = 2n$. Let $S = \{x_1', x_2', \dots, x_n'\}$. Then one can easily verified that there are some geosig path in S which contains all the remaining vertices of $S'(C_n)$. That is if $x_1', x_2', \dots, x_n' \in S$, then $x_1, x_2, \dots, x_n, x_1', x_2', \dots, x_n' \in L[S]$. Thus $L[S] = V\big(S'(C_n)\big)$. Therefore S is a signal set of $S'(C_n)$ and so $sn\big(S'(C_n)\big) \leq |S| = n$. Moreover, we remove any vertex from the set S, then that S is not exists a signal set of cardinality less than that S. Therefore, that S is a minimum signal set of $S'(C_n)$ and hence $sn\big(S'(C_n)\big) = n$.

Theorem 2.8 For any integer $n \geq 4$, $sn\left(S'\left(W_{1,n-1}\right)\right) = n + 1$.

Proof.

Let $x_1, x_2, \dots, x_{n-1}$ be the rim vertices of $W_{1,n-1}$ and x be the apex vertex of $W_{1,n-1}$. Let $x_1', x_2', \dots, x_n'$ be the duplicated vertices of the corresponding vertices $x_1, x_2, \dots, x_n$ respectively in $S'\big(K_{1,n-1}\big)$. Also x' be the duplicated vertex of x and thus the graph $S'\big(W_{1,n-1}\big)$ is obtained, with vertex set $V(S'\big(W_{1,n-1}\big)) = \{x_1, x_2, \dots, x_{n-1}, x', x_1', x_2', \dots, x_{n-1}'\}$. It is clearly that $S'\big(W_{1,n-1}\big)$ has no extreme vertices. Also we observe that if v_i, v_{i+2} for $1 \leq i \leq n-3$ are in a signal set, then there is some geosig path that include the vertices x_1' and x_1. Thus any geosig path between any two non-adjacent vertices in $\{x_1, x_2, \dots, x_{n-1}\}$ contains minimum two vertices. Also $\deg(x) = 2n - 2$ and $degx' = n - 1$ in $S'\big(W_{1,n-1}\big)$, that x and x' does not lie on any geosig between vertices from $\{x_1, x_2, \dots, x_{n-1}\}$. Therefore x and x' must include in every signal set of $S'\big(W_{1,n-1}\big)$. Let $S = x_1, x_2, \dots, x_{n-1}, xx'$. Then L[S]=$V\left(S'\left(W_{1,n-1}\right)\right)$. Furthermore, if we remove any vertex from the set S, then S is not a signal set of $S'(C_n)$. Also, there does not exist a signal set of cardinality lessthan S. Therefore that S is a minimum signal set of cardinality $n + 1$.

Hence $sn\left(S'\left(W_{1,n-1}\right)\right) = n + 1$.

Theorem 2.8 For any integer $n \geq 4$, $sn\left(S'\left(F_{1,n-1}\right)\right) = n + 1$.

Proof.

Let $x_1, x_2, \dots, x_{n-1}$ be the n vertices of $F_{1,n-1}$, where x be the apex vertex and the graph is given in Figure 2.7. Now let as add $x', x_1', x_2', \dots, x_{n-1}'$ be the corresponding duplicated vertices of $x, x_1, x_2, \dots, x_n$ respectively to obtained the graph $S'\big(F_{1,n-1}\big)$.

Clearly, $V(S'\big(F_{1,n-1}\big)) = \{x, x', x_1, x_2, \dots, x_{n-1}, x_1', x_2', \dots, x_{n-1}'\}$ and so $\left|V(S'\big(F_{1,n-1}\big))\right| = 2n$. Then by Theorem 1.1 x_1', x_{n-1}' must be in every signal set of

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$S'(F_{1,n-1})$ because they are the extreme vertices. But it is clear that $S = \{x_1', x_{n-1}'\}$ is not a signal set of $S'(F_{1,n-1})$ itself. Therefore we must add some more vertices in S to obtained the signal set of $S'(F_{1,n-1})$. Let $x_2', x_3', \dots, x_{n-2}' \in S$. Then $x_1, x_2, \dots, x_{n-1}, x_1', x_2', \dots, x_{n-1}' \in L[S]$. But x and x' does not we on any geosig between vertices from S. Thus x and x' must be include in every signal set of $S'(F_{1,n-1})$ and so $sn\left(S'(F_{1,n-1})\right) \geq |S| = n + 1$. But it is clear that $S = \{x, x', x_1', x_2', \dots, x_{n-1}'\}$ itself from a signal set of $S'(F_{1,n-1})$ and so $sn\left(S'(F_{1,n-1})\right) \leq |S| = n + 1$.

Hence $sn\left(S'(F_{1,n-1})\right) s = n + 1$.

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