
STUDY OF INVENTORY MODEL OF DETERIORATING PRODUCT SHORTAGE

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Abstract: The study is focuses on the market's shortage of successful products. The products whose shortage is examined in this research article are foods, vegetables, and fruits. When the customer requests it, the item is not immediately available. There are numerous causes behind the market's current product deficit. The cost of production stoppage, overtime or idle time payments, special orders at the higher price, idle machine loss of goodwill, loss of opportunity to sell, and loss of profitability are discussed in this research paper. The shortage may be from the manufacturer's side. It may be of cost of backlogging and may be of cost of no backlogging. The shortage may be from the seller's side. They are resolving this mathematical issue with the aid of an economic order quantity mathematical model.

Key word: economic order quantity, mathematical model, Foods, vegetables, and fruits.

Introduction: Since 65% of the population depends on agriculture for their livelihood, it is one of the key sectors of the Indian economy. Food security is a concern for everyone, but it is especially important for populous nations like India and China. India holds the top spot both in terms of extent and value among the nations that rely on rain-fed agriculture.

When the buyer requests it, the item is not always made available. The analysis focuses on the market's shortage of successful products foods, fruits, and veggies. The elements of shortfall production include the price of production interruption, overtime or idle time payments, special orders at higher prices, idle machine loss of goodwill, loss of opportunity to sell, and loss of profitability.

To resolve this mathematical issue, use the economic order quantity mathematical model. Order number is the number of materials or goods that a company produces or purchases throughout a manufacturing cycle. Economic Order Quantity is the Order Quantity for which the Total Cost per period is minimum.



The goal of the essay is to examine the type and scale of cereal production in India from 1950 onwards as well as the impact of rainfall on per capita availability of vegetables, fruits, and food. As these inputs were discovered to have a considerable impact on grain production, we also included additional influencing elements in the model, such as the amount of land utilized for cereal production, the proportion of the area covered by irrigation, and the application of fertilizer.

We used several predictive categorization models that can anticipate vegetables, fruits, and food shortages from the monsoon rainfall and other influencing factors to predict the likelihood of a shortage in advance, right after the monsoon period. The purpose of the research presented in this paper is to propose a methodology based on machine learning algorithms, and more specifically based on binary classifiers of data mining jobs, that can forecast vegetables, fruits, and food shortage specified in the problem. Due to the nature of the application, where the prediction of a shortage of vegetables, fruits, and food is a complex process with sufficient interacting variables, machine learning technologies have been used.

Data source: The research, which spans the years from the fiscal year 1950–51 to 2020–21, is conducted using data that are readily available from reliable sources. Data on India's population was retrieved from the World Bank website (<http://databank.worldbank.org/>) and the Indian government's data portal (<https://data.gov.in/>) to calculate per capita consumption of vegetables, fruits, and food. Producing vegetables, fruits, and crops as well as obtaining other agricultural inputs came from the "Database of Indian Economy" that the Reserve Bank of India made available. (<http://dbie.rbi.org.in/DBIE/dbie.rbi?site=home>).

Year	Population
1950-51	376,325,200

1960-61	449,480,608
1970-71	553,578,513
1980-81	696,783,517
1990-91	870,133,480
2000-01	1,015,974,042
2010-11	1,203,098,691
2020-21	1,384,845,360

In 1950–1951, there were 376,325,200 people living in the country. The population at this time was 449, 4890, 608 in 1960–1961. Once more, the population at that time was 553,578,513 in the following ten years, from 1970 to 1971. Following that, India's population increased throughout the course of the following ten years, reaching a peak of 696,783,517 people in 1980–1981. The entire population of India in the ensuing ten years, or from 1990 to 1991, was 870,133,480. Again, in the following ten years, India's population would reach 1,015,974,042 in 2000-2001. There were 1,203,098,691 people living in India in 2010–11.

The population of India will be 1,384,845,360 in 2020.

Notation and assumptions: EOQ concept that allows for shortages

Let's indicate that this is the deterministic EOQ model by:

N: number of orders placed per year

Q: number of units ordered

D: demand in units of inventory per year

C: cost of purchase

C_o : ordering cost per order

C_h : holding cost per unit per unit per period of the inventory

L: lead time

R: re-order point

r_b : Replacement rate which lot size Q is ordered to the inventory

t: time between the placement of two successive orders which is expressed as a factorial part of the standard time horizon

T_c : total inventory cost

The model is explained considering the following situations:

Demand is always present and known,

Cost of Purchase is always present and known,

Lead Time is zero,

Ordering Cost is always present and known,

Shortage is permitted, and Replenishment Rate is instantaneous.

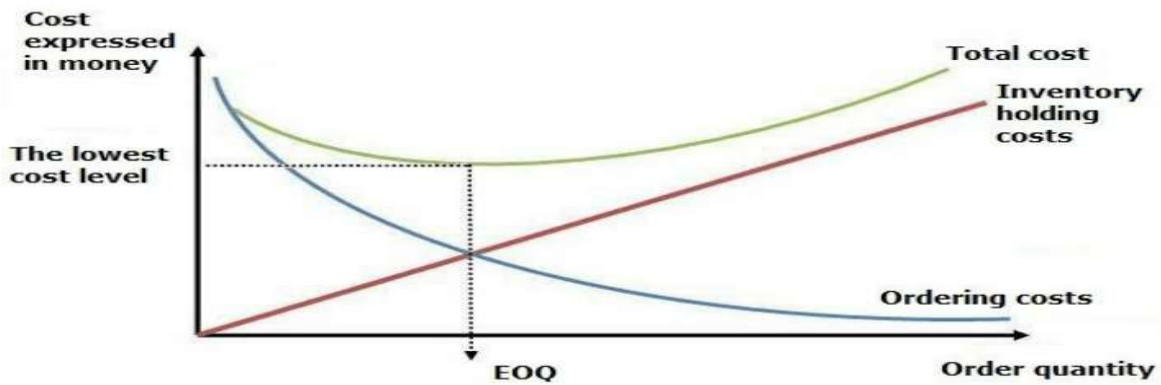
Methodology: Economic Order Quantity (EOQ) is a crucial inventory management tool that shows how much of an item is available to lower the overall cost of handling inventory (Handling Cost) and processing orders (Ordering Cost). The goal of establishing the EOQ is to minimize the Total Incremental Cost, which includes two major total costs: the total cost of ordering the goods

and the total cost of handling it. The mathematical model is emphasized as being quite important to improve the inventory management of a product in this study work, which contextually highlights two fundamental approaches of finding the EOQ: the trial-and-error method and the mathematical approach.

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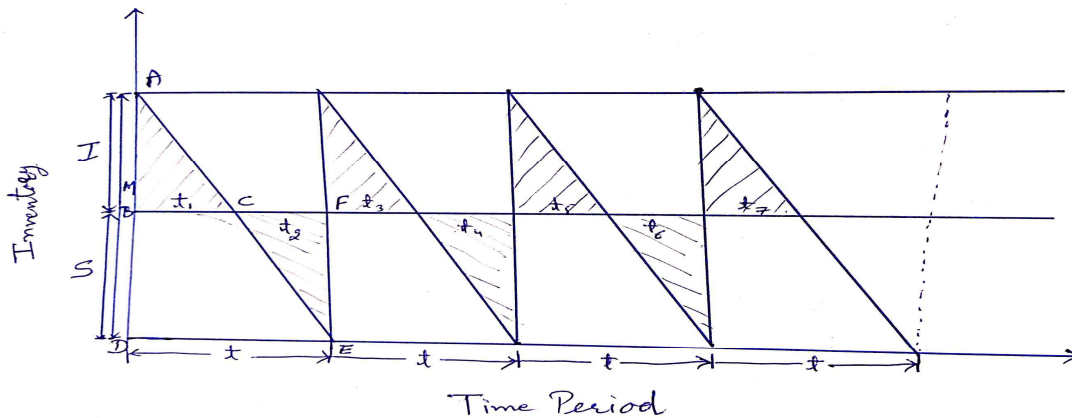
To resolve this mathematical issue, use the economic order quantity mathematical model. Order number is the number of materials or goods that a company produces or purchases throughout a manufacturing cycle. Economic Order Quantity is the Order Quantity for which the Total Cost per Period is Minimum.

As a result, EOQ is the amount of order size that minimizes total cost (cost of carrying inventory plus ordering cost).



The cost of the shortage is assumed to be precisely proportional to the number of units shorted out in this model, with the exception that shortages are permitted and occur frequently. Below is a diagram that illustrates instantaneous supply:

When Shortages are Permitted, the EOQ Model



If the inventory level is between the order quantity Q and reorder point R and is denoted by I and O to R is the shortage level and is marked by S , then the upper and lower triangles here are reflecting the inventory and shortage.

Moreover, the cycle duration is $t = t_1 + t_2$

The order placement through order receipt is the cycle period. The time for holding the inventory is indicated by the order point to order placement t_1 and the time between an order being placed and an order being received is indicated by t_2 .

Let θ be the quantity so that $\theta = I + S$

Different inventory costs are provided by:

$$\text{Ordering cost (C): } \frac{C_0 D}{Q}$$

$$\text{Average Inventory} = \frac{I}{2} \text{ or } \frac{Q-S}{2}$$

$$\text{Carrying cost (C.C.)} = C_r \frac{I}{2} t_1$$

Given that ADE and CEF are similar triangles

$$\text{So, } \frac{t_1}{t} = \frac{I}{Q} \rightarrow t_1 = \frac{I}{Q} t$$

$$\text{C.C.} = \frac{C_r I}{2} \frac{I}{Q} t = C_r \frac{I^2}{2Q} t$$

If $t = 1$ year

$$\text{Then C.C.} = \frac{C_r I^2}{2Q}$$

Shortage cost (S.C.) = (shortage cost per unit per period) \times (Av. Number of units short out) \times (period of shortage per second)

$$\text{That is S.C.} = C_s \frac{S}{2} t_2 = C_s \frac{S}{2} (t - t_1)$$

Given that ADE and CEF are similar triangles

$$\frac{t_2}{t} = \frac{S}{Q} \rightarrow t_2 = \frac{S}{Q} t$$

$$\text{So, S.C.} = C_s \frac{S}{2} \frac{S}{Q} t = C_s \frac{S^2}{2Q} t$$

Now total cost (Annual relevant cost)

$$TC = CD + \frac{C_0 D}{Q} + \frac{C_h I^2}{2Q} + \frac{C_s S^2}{2Q} \quad (2.1)$$

Where CD is fixed cost, $\frac{C_0 D}{Q} + \frac{C_h I^2}{2Q}$ is variable cost, $\frac{C_s S^2}{2Q}$ is shortage cost

in where $S = Q - I$

By first partially differentiating (2.1) with respect to I and then again with respect to Q , and then equating the equation to zero, the optimal values of Q and I are discovered.

$$\frac{d}{dI} (TC) = 0 + 0 + \frac{2C_h I}{2Q} + \frac{d}{dI} \left(\frac{C_s (Q - I)^2}{2Q} \right)$$

$$0 = \frac{2C_h I}{2Q} + -2 \left(\frac{C_s (Q - I)}{2Q} \right)$$

$$C_h I = C_s (Q - I)$$

$$(C_h + C_s)I = C_s Q$$

$$I = \frac{C_s Q}{C_h + C_s} \quad (2.2)$$

Once more differentiating (2.1) in relation to Q, we obtain

$$\frac{d}{dQ} (\text{T. C.}) = 0 - \frac{C_0 D}{Q^2} - \frac{2C_h I^2}{2Q^2} + \frac{d}{dQ} \left(\frac{C_s(Q - I)^2}{2Q} \right)$$

$$0 = -\frac{C_s D}{Q^2} - \frac{2C_h I^2}{2Q^2} + \frac{d}{dQ} \left(\frac{C_s}{2} \left(\frac{Q^2 + I^2 - 2QI}{Q} \right) \right)$$

$$0 = -\frac{C_s D}{Q^2} - \frac{2C_h I^2}{2Q^2} + \frac{d}{dQ} \left(\frac{C_s}{2} \left(Q + \frac{I^2}{Q} - 2I \right) \right)$$

$$0 = -\frac{C_s D}{Q^2} - \frac{C_h I^2}{Q^2} + \frac{C_s}{2} Q - \frac{C_s I^2}{2Q^2}$$

$$\frac{2C_s D + 2C_h I^2 + 2C_s I^2}{2Q^2} = \frac{C_s}{2}$$

$$Q^2 = \frac{C_s}{2C_s D + 2I^2 (C_h + C_s)} \quad (2.3)$$

Based on equation (2.2),

$$Q = \frac{(C_h + C_s)}{C_s} I$$

Equation (2.3) then changes to

$$\left[\frac{(C_h + C_s) I}{C_s} \right]^2 = \frac{C_s}{2C_s D + 2I^2 (C_h + C_s)}$$

$$Q^* = \sqrt{\left(\frac{2DC_0}{C_r} \right) \left(\frac{C_h + C_s}{C_s} \right)}$$

And

$$I^* = \sqrt{\left(\frac{2DC_0}{C_h} \right) \left(\frac{C_s}{C_h + C_s} \right)}$$

The following other traits are:

$$S^* = Q^* - I^*$$

Cycle period: $\frac{Q^*}{D} = t^*$

that is $t^* = \frac{1}{D} \sqrt{\left(\frac{2DC_0}{C_h} \right) \left(\frac{C_h + C_s}{C_s} \right)}$

Number of orders per cycle period

$$N^* = \frac{1}{t^*} \text{ or } \frac{D}{t^*}$$

And the minimum total cost is $T.C^* = CD + \frac{C_0 D}{Q^*} + \frac{C_h I^{*2}}{2Q^*} + C_s \frac{S^{*2}}{2Q^*}$

$$\text{Also, } T.C^* = CD + \sqrt{2 C_0 C_r \left(\frac{C_s}{C_h + C_s} \right) D}$$

Development of model: If the inventory level is between the order quantity Q and reorder point R and is denoted by I and O to R is the shortage level and is marked by S , then the upper and lower triangles here are reflecting the inventory and shortage.

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$$TC = CD + \frac{C_0 D}{Q} + \frac{C_h I^2}{2Q} + \frac{C_s S^2}{2Q} \quad (3.1)$$

Where CD is fixed cost, $C_0 D + \frac{C_h I^2}{2Q}$ is variable cost, $\frac{C_r S^2}{2Q}$ is shortage cost

Where $S = Q - I$

The optimum value of Q and I are obtained by first partially differentiating (3.1) with respect to I and again with respect to Q and equating the equation to zero.

$$\frac{d}{dI} (TC) = 0 + 0 + \frac{2C_r I}{2Q} + \frac{d}{dI} \left(\frac{C_s (Q - I)^2}{2Q} \right)$$

$$0 = \frac{2C_h I}{2Q} + -2 \left(\frac{C_s(Q - I)}{2Q} \right)$$

$$C_h I = C_s(Q - I)$$

$$(C_h + C_s)I = C_s Q$$

$$I = \frac{C_s Q}{C_h + C_s} \quad (3.2)$$

Again differentiating (3.1) with respect to Q, we get

$$\frac{d}{dQ} (\text{T.C.}) = 0 - \frac{C_0 D}{Q^2} - \frac{2C_h I^2}{2Q^2} + \frac{d}{dQ} \left(\frac{C_s(Q - I)^2}{2Q} \right)$$

$$0 = -\frac{C_s D}{Q^2} - \frac{2C_h I^2}{2Q^2} + \frac{d}{dQ} \left(\frac{C_s}{2} \left(\frac{Q^2 + I^2 - 2QI}{Q} \right) \right)$$

$$0 = -\frac{C_s D}{Q^2} - \frac{2C_h I^2}{2Q^2} + \frac{d}{dQ} \left(\frac{C_s}{2} \left(Q + \frac{I^2}{Q} - 2I \right) \right)$$

$$0 = -\frac{C_s D}{Q^2} - \frac{C_h I^2}{Q^2} + \frac{C_s}{2} Q - \frac{C_s I^2}{2Q^2}$$

$$\frac{2C_s D + 2C_h I^2 + 2C_s I^2}{2Q^2} = \frac{C_s}{2}$$

$$Q^2 = \frac{C_s}{2C_s D + 2I^2 (C_h + C_s)} \quad (3.3)$$

From equation (3.2)

$$Q = \frac{(C_h + C_s)}{C_s} I$$

So, equation (3.3) becomes

$$\left[\frac{(C_h + C_s) I}{C_s} \right]^2 = \frac{C_s}{2C_s D + 2I^2 (C_h + C_s)}$$

$$Q^* = \sqrt{\left(\frac{2DC_0}{C_r} \right) \left(\frac{C_h + C_s}{C_s} \right)}$$

And

$$I^* = \sqrt{\left(\frac{2DC_0}{C_h} \right) \left(\frac{C_s}{C_h + C_s} \right)}$$

Now the other characteristics are:

$$S^* = Q^* - I^*$$

$$\text{Cycle period: } \frac{Q^*}{D} = t^*$$

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Also, $T.C^* = CD + \sqrt{2 C_0 C_r \left(\frac{C_s}{C_h + C_s} \right) D}$

Conceptual analysis of result: We tried to predict the production of vegetables, fruits, and food shortage throughout the year. To do this, we used per capita availability of vegetables, fruits, and food as our objective variable and compared it to the anticipated availability of those items. Using the data from independent variables, irrigation area, area used to produce vegetables, fruits, and food, and fertilizer, we can easily predict the potential for a shortage of vegetables, fruits, and food in our nation. This is an interesting observation that directly benefits policymakers, the government, and related businesses. The choice of acceptable methodology and the discovery of characteristics that can be used to forecast the occurrence of vegetable, fruit, and food shortages are thus the study's main contributions.

Conclusion: Understanding how vegetables, fruits, and food are produced in India was the paper's main goal. It is common knowledge that changes in production levels can have a significant impact on the nation's agricultural output as well as the production of fruits, vegetables, and food. To forecast the production of vegetables, fruits, and food shortage throughout the year, we used per capita availability of vegetables, fruits, and food as our target variable. We then compared the projected availability of vegetables, fruits, and food to the average per capita availability. Using the data from independent variables, irrigation area, and area, policymakers, the government, and linked enterprises stand to gain directly from this intriguing insight. We may estimate the potential lack of veggies, fruits, and food in our nation with ease by considering fertilizer, food production, and fruits. The choice of acceptable methodology and the discovery of characteristics that can be used to forecast the occurrence of vegetable, fruit, and food shortages are thus the study's main contributions. The use of further data mining models and the incorporation of additional influencing factors that may have an impact on the production of fruits, vegetables, and other food items are both included in the scope of future study.

Result: One of the top concerns of policymakers and the government is ensuring the security of vegetables, fruits, and food given that the country with a measly 2% of land resources supplies 17% of the world's population's needs for food. The process becomes challenging because the production of fruits, vegetables, and other foods depends heavily on rain during the monsoon season. The study offers a framework that can forecast per-person production of fruits, vegetables, and food. In addition, this kind of information can be used by politicians, the government, and the agricultural industry to make planned choices and act if there is an unanticipated drop in the production of fruits, vegetables, and food.

Conflict of result: The authors do not have any conflict of interest.

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