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SUSTAINABLE INVENTORY MODEL FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH THE EFFECT OF GREEN TECHNOLOGY AND INFLATION.

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Abstract

Managing inventory in a sustainable and cost-effective manner is a critical challenge for organizations, particularly when dealing with non-instantaneous deteriorating items in the presence of inflation. This study presents a sustainable inventory model with three different situations. In the first situation inventory model with a carbon tax policy without investment in green technology is considered. In second sustainable inventory model with carbon tax policy with investment in green technology. In the third situation sustainable inventory model with carbon cap-and trade policy with investment in green technology. A numerical illustration is carried out by using the software Mathematica 12.0. A key focus of this model is the incorporation of green technology practices, which can significantly reduce the environmental footprints associated with inventory management. The results shows that we get maximum profit in the third situation. To validate the model sensitivity analysis is carried out for various parameters which shows positive and negative impact on optimal results.

Keywords: Green Technology, Carbon Tax policy, Carbon Cap-and Trade, inflation, non-instantaneous deterioration and variable holding cost.

1. Introduction

Managing inventory effectively is a critical concern for businesses across various industries, as it directly impacts their operational efficiency, financial performance, and environmental sustainability. In today's world, where environmental concerns and economic factors are becoming increasingly intertwined, the need for sustainable inventory models has gained significant attention. One such area of focus is the management of non-instantaneous deteriorating items, which are products that experience deterioration or spoilage over time but not immediately. In this complex landscape, the integration of green technology and the consideration of inflation further add to the intricacy of inventory management.

managing non-instantaneous deterioration in inventory requires specialized inventory models and strategies that consider factors such as the rate of deterioration, shelf life, and quality of items over time. These models and strategies help businesses optimize their inventory decisions while minimizing losses due to deterioration or obsolescence.

To effectively manage inventory in an inflationary environment, businesses need to consider the impact of rising prices on their costs, pricing strategies, and reorder points. Many companies use

sophisticated inventory models that incorporate inflation into their calculations to make informed decisions about when and how much to order.
Variable holding costs are an important consideration in inventory management. Businesses need to calculate and manage these costs to make informed decisions about order quantities, storage practices, and overall inventory strategy. Reducing variable holding costs can contribute to cost savings and improved inventory management efficiency.
Incorporating carbon tax and cap-and-trade considerations into inventory management requires a holistic approach. It involves evaluating the environmental impact of various inventory decisions, assessing the potential financial implications of carbon policies, and making strategic choices that align with both sustainability and cost-efficiency goals.

2. Literature Review

In today's rapidly evolving business landscape, inventory control is a critical aspect of supply chain operations that extends far beyond mere cost control and logistics optimization. It has evolved into a multifaceted discipline that must also address sustainability concerns, technological advancements, and the economic impact of factors such as inflation. This complexity is particularly pronounced when dealing with non-instantaneous deteriorating items, where the items in stock gradually degrade over time and cannot be stored indefinitely. Moreover, the integration of green technology adds an additional layer of complexity and responsibility to inventory management. Ghare and Scharder (1963) was the first who gave the concept of deterioration in inventory modeling. After that Covert and Philip (1973) extended the model with time-depend deterioration. Buzacott (1975) first gives the concept of inflation in the inventory modeling. Kumar et al (2010) developed production model in which demand is taken time dependent. The also consider the effect of shortage which is partially backlogged. Singh et al (2011) formulate a two-warehouse inventory model for deteriorating items. In which demand is taken stock dependent, they also consider the effect of inflation and shortages. Cap and trade were initially used in the inventory model by Hua et al. (2011). After that, Jaber et al. (2013) created the first investment model for cutting carbon emissions. In their 2013 article, Benjaafar et al. covered the continuous SC policy with a carbon footprint. Wee et al (2005) developed two warehouse inventory model that consider Weibull distribution deterioration partial backordering and inflation. Qin et al (2015) they formulate sustainable trade credit model that considers replenishment policies under carbon cap and trade and carbon tax regulations. Shaikh et al (2017) formulate non-instantaneous deteriorating inventory model. In which demand is demand being depends on price and stock level. They also consider shortage and inflation. Shah & Naik (2018) developed inventory model with non-instantaneous deterioration. They also consider learning effect and price dependent demand. Pal and Samanta (2018) developed inventory model for non-instantaneous deteriorating items. They also consider random pre-deterioration period. Rastogi and Singh (2018) developed a production model for decaying items. In which effect of shortage and inflation is taken into consideration. Lu et al (2020) they give a Stackelberg gaming approach for sustainable production inventory model. Khanna et al (2020) showed the effect of carbon tax and carbon cap and trade policy for inventory system. In which they also consider price dependent demand and preservation technology. Damyad & Jafari (2021) developed a three-echelon inventory model for decaying items. They codify this model and also consider inflation and shortages. Sun & Yang (2021) gave the optimal decisions for competitive manufacturer. They use carbon tax and cap-and-trade policy to reducing carbon emission. Singh et al (2022) they formulate an inventory model for buyer in which they consider the impact of green technology, shortage and inflation. Dharmesh & Kumal (2022) formulate a sustainable inventory model that considers price sensitive demand with expiration date and they investment in green technology. Shah et al (2023) developed sustainable production inventory model they use green technology to reducing GHG emission. Kumar et al (2023) developed a model for retailer in which they showed the joint effect of selling price and promotional efforts on inventory control. They also considers effect of trade credit, inflation, variable holding cost and partial backlogging.

From the above literature we conclude that there is gap in the research sustainable inventory model with non-instantaneous deterioration with variable holding cost, partial backlogging and effect of inflation under different carbon emission policies. To incorporate all the factors, mention above we develop sustainable inventory model in three different situations.

i. Sustainable inventory model with carbon tax policy without green technology investment.
ii. Sustainable inventory model with carbon tax policy with green technology investment.
iii. Sustainable inventory model with carbon cap and trade policy with green technology investment.

3. Assumptions and Notations

3.1 Assumptions

1. Single items are considered in this model.
2. Deterioration is non-instantaneous.
3. Effect of inflation is taken into consideration
4. Holding cost is taken as time varying.
5. Green technology is used.
6. The planning horizon is infinite.
7. Carbon emission occurred from holding, ordering and purchased unit are considered.

3.2 Notations

Table 1 Notation used throuout in the models.

Parameters	Explanation
D	Demand rate
Q	Order Quantity
β	Backlogging rate
θ	Rate of deterioration
A	Order Quantity

h, g	Holding cost parameters
C_d	Deterioration Cost per unit per unit time
C_s	Shortage cost per unit per unit time
C_l	Lost sale cost
c	Purchasing cost
O_{CE}	The fixed carbon emission per unit of order
C_E	Carbon emission cost per purchased unit
h_{CE}	Carbon emission cost in holding process
C_T	Carbon tax per carbon emission unit
C_p	Under the carbon cap-and-Trade policy, the carbon price per carbon emission
C_c	Carbon cap was given under the carbon cap-and trade policy.
r	Rate of inflation
t_1	Time when deterioration start
Decision Variables	
t_2	Time at which the inventory level becomes zero
T	Cycle Length

4. Mathematical Formulation

This model is developed for the retailing business. In the beginning at time $t = 0$ the inventory level is maximum Q_1. In the time interval $0 \leq t \leq t_1$ the products are as good as fresh so in this interval there is no deterioration. Inventory level depletes only due to demand. After that at time t_1 deterioration starts. In the time interval, $t_1 \leq t \leq t_2$ inventory level depletes due to joint effect of demand and deterioration. At time $t = t_2$inventory level become zero and shortages occur in the interval $t_2 \leq t \leq T$.

In the interval $[0, t_1]$ the governing differential equation is given by

$$\dot{I}_1(t) = -D, \ \ 0 < t \leq t_1 \qquad (1)$$

With boundary condition$I_1(0) = Q_1$

The inventory level is given by

$$I_1(t) = -Dt + Q_1, 0 \leq t \leq t_1 \qquad (2)$$

In the interval $[t_1, t_2]$ the governing differentiatial equation is given by

$$\dot{I}_2(t) = -D - \theta I(t), \ \ t_1 < t \leq t_2 \qquad (3)$$

With boundary condition $I_2(t_2) = 0$

The inventory level is given by

$$I_2(t) = \frac{D}{\theta}\left(e^{\theta(t_2 - t)} - 1\right), \qquad t_1 \leq t \leq t_2 \qquad (4)$$

From the continuity condition at $t = t_1$, $I_1(t_1) = I_2(t_2)$ we get

$$Q_1 = Dt_1 + \frac{D}{\theta}(e^{\theta(t_2-t_1)} - 1) \tag{5}$$

In the interval $[t_2, T]$ the governing differential equation is given by

$$\dot{I}_3(t) = -D\beta, \;\; t_2 < t \leq T \tag{6}$$

With boundary condition $I_3(t_2) = 0, I_3(T) = -B$

The inventory level is given by

$$I_3(t) = D\beta(t_2 - t) \tag{7}$$

Now the backorder quantity is given by

$$B = D\beta(T - t_2) \tag{8}$$

The order quantity is given by

$$Q = Q_1 + B$$

$$Q = Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right) + D\beta(T - t_2) \tag{9}$$

Costs Components

1. Ordering Cost = A
2. Holding Cost = $\int_0^{t_1}(h + gt)e^{-rt}I_1(t)dt + \int_{t_1}^{t_1}(h + gt)\, e^{-r}\; I_2(t)dt$

$$\text{Holding Cost} = D[\frac{-ht_1e^{-rt_1}}{r} - \frac{he^{-rt_1}}{r^2} + \frac{h}{r^2} + g\left(\frac{-t_1^2e^{-rt_1}}{r} - \frac{2t_1e^{-rt_1}}{r^2} - \frac{2e^{-rt_1}}{r^3} + \frac{2}{r^3}\right) + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{-(h+gt_1)e^{-rt_1}}{r} + \frac{h}{r} - \frac{ge^{-rt_1}}{r^2} + \frac{g}{r^2}\right] + \frac{D}{\theta}\left[\frac{-(h+gt_2)e^{-rt_2}}{(\theta+r)} - \frac{ge^{-rt_2}}{(\theta+r)^2} + \frac{(h+gt_1)e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{ge^{\theta(t_2-t_1)-rt_1}}{(\theta+r)^2}\right] - \frac{D}{\theta}[\frac{-(h+gt_2)e^{-rt_2}}{r} + \frac{(h+gt_1)e^{-rt_1}}{r} - \frac{ge^{-rt_2}}{r^2} + \frac{ge^{-rt_1}}{r^2}] \tag{10}$$

3. Deterioration Cost

$$DC = C_d\int_{t_1}^{t_2}\theta I_2(t)e^{-rt}dt$$

$$DC = C_dD[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}}{r} - \frac{e^{-rt_1}}{r}] \tag{11}$$

4. Shortage Cost

$$SC = -C_s\int_{t_2}^{T} I_3(t)e^{-rt}\, dt$$

$$SC = -C_s\beta D[\frac{(t_2-T)e^{-r}}{r} + \frac{e^{-rt_2}}{r^2} - \frac{e^{-rT}}{r^2}] \tag{12}$$

5. Lost Sale Cost = $C_i\int_{t_2}^{T}(1 - \beta)De^{-rt}dt$

$$LSC = \; C_i(1 - \beta)D[\frac{-e^{-rT}}{r} + \frac{e^{-rt_2}}{r}]$$

6. Purchasing Cost = cQ

$$PC = c[Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right) + D\beta(T - t_2)] \quad (13)$$

7. Carbon Emission Cost

CEC= $O_{CE} + C_{PE}Q + h_{CE}\{\int_0^{t_1} I_1(t)e^{-r}\ dt + \int_{t_1}^{t_2} I_2(t)e^{-rt}dt\}$

CEC= $O_{CE} + C_{PE}Q + h_{CE}\{D\left[\frac{t_1e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{1}{r^2}\right] + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{1-e^{-rt_1}}{r}\right] + \frac{D}{\theta}\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}-e^{-rt_1}}{r}\right]\}$ (14)

8. Sales Revenue

$$SR = p\int_0^{t_2} Ddt + pB$$

$$SR = pDt_2 + p\beta D(T - t_2) \quad (15)$$

4.1 Model formulation under the carbon tax policy without green investment

Total profit under the carbon tax policy without green Technology

$$TP_1 = \frac{1}{T}[SR - PC - OC - HC - DC - SC - LSC - C_TCEC]$$

$$\begin{aligned} TP_1 = \frac{1}{T}[&pDt_2 + p\beta D(T - t_2) - c\left[Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right) + D\beta(T - t_2)\right] - A \\ &- D\left[\frac{-ht_1e^{-rt_1}}{r} - \frac{he^{-rt_1}}{r^2} + \frac{h}{r^2} + g\left(\frac{-t_1^2e^{-rt_1}}{r} - \frac{2t_1e^{-rt_1}}{r^2} - \frac{2e^{-rt_1}}{r^3} + \frac{2}{r^3}\right) \right.\\ &+ Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{-(h+gt_1)e^{-rt_1}}{r} + \frac{h}{r} - \frac{ge^{-rt_1}}{r^2} + \frac{g}{r^2}\right] \\ &+ \frac{D}{\theta}\left[\frac{-(h+gt_2)e^{-rt_2}}{(\theta+r)} - \frac{ge^{-rt_2}}{(\theta+r)^2} + \frac{(h+gt_1)e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{ge^{\theta(t_2-t_1)-rt_1}}{(\theta+r)^2}\right] \\ &- \frac{D}{\theta}\left[\frac{-(h+gt_2)e^{-rt_2}}{r} + \frac{(h+gt_1)e^{-rt_1}}{r} - \frac{ge^{-rt_2}}{r^2} + \frac{ge^{-rt_1}}{r^2}\right] \\ &- C_dD\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}}{r} - \frac{e^{-rt_1}}{r}\right] \\ &+ C_s\beta D\left[\frac{(t_2-T)e^{-rT}}{r} + \frac{e^{-rt_2}}{r^2} - \frac{e^{-rT}}{r^2}\right] - C_i(1-\beta)D\left[\frac{-e^{-rT}}{r} + \frac{e^{-rt_2}}{r}\right] \\ &- C_T[O_{CE} + C_{PE}Q \\ &+ h_{CE}\left\{D\left[\frac{t_1e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{1}{r^2}\right] + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{1-e^{-rt_1}}{r}\right] \right.\\ &\left.\left. + \frac{D}{\theta}\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}-e^{-rt_1}}{r}\right]\right\}\right]] \end{aligned}$$

4.2 Model formulation under the carbon tax policy without green investment

This model operates under carbon tax policy with green investment. Hence, the green investment. Hence, the green investment is $GI = \frac{aG^2T}{2}$

The minimum threshold value of carbon emission units under green investment is

$$CU_1 = k\,CEC = k[O_{CE} + C_{PE}Q + h_{CE}\{D\left[\frac{t_1 e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{1}{r^2}\right] + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{1-e^{-rt_1}}{r}\right] + \frac{D}{\theta}\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}-e^{-rt_1}}{r}\right]\}]$$

Effective carbon emission level under green investment is

$$CUG = CU_1 + (CEC - CU_1)e^{-aG}$$

$$CUG = [O_{CE} + C_{PE}Q + h_{CE}\{D\left[\frac{t_1 e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{1}{r^2}\right] + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{1-e^{-rt_1}}{r}\right] + \frac{D}{\theta}\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}-e^{-rt_1}}{r}\right]\}](k + (1-k)e^{-aG}$$

The total emission cost is $EC = C_T CUG$

$$EC = C_T[O_{CE} + C_{PE}Q + h_{CE}\{D\left[\frac{t_1 e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{1}{r^2}\right] + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{1-e^{-rt_1}}{r}\right] + \frac{D}{\theta}\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}-e^{-rt_1}}{r}\right]\}](k + (1-k)e^{-a}$$

Thus, the total profit under the carbon tax policy with green investment is

$$TP_2 = \frac{1}{T}[SR - PC - OC - HC - DC - SC - LSC - GI - EC]$$

$$\begin{aligned}
TP_2 = \frac{1}{T}&[pDt_2 + p\beta D(T-t_2) - c\left[Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right) + D\beta(T-t_2)\right] - A \\
&- D\left[\frac{-ht_1e^{-rt_1}}{r} - \frac{he^{-rt_1}}{r^2} + \frac{h}{r^2} + g\left(\frac{-t_1^2e^{-rt_1}}{r} - \frac{2t_1e^{-rt_1}}{r^2} - \frac{2e^{-rt_1}}{r^3} + \frac{2}{r^3}\right)\right. \\
&+ Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{-(h+gt_1)e^{-rt_1}}{r} + \frac{h}{r} - \frac{ge^{-rt_1}}{r^2} + \frac{g}{r^2}\right] \\
&+ \frac{D}{\theta}\left[\frac{-(h+gt_2)e^{-rt_2}}{(\theta+r)} - \frac{ge^{-rt_2}}{(\theta+r)^2} + \frac{(h+gt_1)e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{ge^{\theta(t_2-t_1)-rt_1}}{(\theta+r)^2}\right] \\
&- \frac{D}{\theta}\left[\frac{-(h+gt_2)e^{-rt_2}}{r} + \frac{(h+gt_1)e^{-rt_1}}{r} - \frac{ge^{-rt_2}}{r^2} + \frac{ge^{-rt_1}}{r^2}\right] \\
&- C_dD\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}}{r} - \frac{e^{-rt_1}}{r}\right] \\
&+ C_s\beta D\left[\frac{(t_2-T)e^{-rT}}{r} + \frac{e^{-rt_2}}{r^2} - \frac{e^{-rT}}{r^2}\right] - C_i(1-\beta)D\left[\frac{-e^{-rT}}{r} + \frac{e^{-rt_2}}{r}\right] - \frac{aG^2T}{2} \\
&- C_T(k + (1-k)e^{aG})[O_{CE} + C_{PE}Q \\
&+ h_{CE}\left\{D\left[\frac{t_1e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{1}{r^2}\right] + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{1-e^{-rt_1}}{r}\right]\right. \\
&\left.\left. + \frac{D}{\theta}\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}-e^{-rt_1}}{r}\right]\right\}\right]]
\end{aligned}$$

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Model 4.3 Formulation of model under cap-and-trade policy with green investment

This model operates under a carbon cap and Trade policy with green investment. Carbon emission cost under this policy is

$$EC_c = -C_p(C_c - CUG)$$

$$EC_c = -C_p(C_c - [O_{CE} + C_{PE}Q + h_{CE}\{D\left[\frac{t_1e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{1}{r^2}\right] + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{1-e^{-rt_1}}{r}\right] + \frac{D}{\theta}\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}-e^{-rt_1}}{r}\right]\}](k + (1-k)e^{-aG})$$

As a result, the total profit under carbon- cap-trade policy with green investment is

$$TP_3 = \frac{1}{T}[pDt_2 + p\beta D(T-t_2) - c\left[Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right) + D\beta(T-t_2)\right] - A - D\left[\frac{-ht_1e^{-rt_1}}{r} - \frac{he^{-rt_1}}{r^2} + \frac{h}{r^2} + g\left(\frac{-t_1^2e^{-rt_1}}{r} - \frac{2t_1e^{-rt_1}}{r^2} - \frac{2e^{-rt_1}}{r^3} + \frac{2}{r^3}\right) + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{-(h+gt_1)e^{-rt_1}}{r} + \frac{h}{r} - \frac{ge^{-rt_1}}{r^2} + \frac{g}{r^2}\right] + \frac{D}{\theta}\left[\frac{-(h+gt_2)e^{-rt_2}}{(\theta+r)} - \frac{ge^{-rt_2}}{(\theta+r)^2} + \frac{(h+gt_1)e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{ge^{\theta(t_2-t_1)-rt_1}}{(\theta+r)^2}\right] - \frac{D}{\theta}\left[\frac{-(h+gt_2)e^{-rt_2}}{r} + \frac{(h+gt_1)e^{-rt_1}}{r} - \frac{ge^{-rt_2}}{r^2} + \frac{ge^{-rt_1}}{r^2}\right] - C_dD\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}}{r} - \frac{e^{-rt_1}}{r}\right] + C_s\beta D\left[\frac{(t_2-T)e^{-rT}}{r} + \frac{e^{-rt_2}}{r^2} - \frac{e^{-rT}}{r^2}\right] - C_i(1-\beta)D\left[\frac{-e^{-rT}}{r} + \frac{e^{-rt_2}}{r}\right] - \frac{aG^2T}{2} + C_p[C_c - (k + (1-k)e^{aG})[O_{CE} + C_{PE}Q + h_{CE}\left\{D\left[\frac{t_1e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{1}{r^2}\right] + Dt_1 + \frac{D}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\left[\frac{1-e^{-rt_1}}{r}\right] + \frac{D}{\theta}\left[\frac{-e^{-rt_2}}{(\theta+r)} + \frac{e^{\theta(t_2-t_1)-rt_1}}{(\theta+r)} + \frac{e^{-rt_2}-e^{-rt_1}}{r}\right]\right\}]]$$

5. Numerical Illustration

Consider a business situation in which input parameters are in appropriate units are given in table 2.

Table 2 Values of input parameters

Parameters	**Values**	**Parameters**	**Values**
D	1500 Units	r	0.5
β	0.2	C_c	200
θ	0.1	C_p	0.5
A	70 $/order	C_T	0.4
h	1$/unit	h_{CE}	2
C_d	1$/unit	t_1	10
C_s	3$	C_{PE}	1
C_l	0.3$	O_{CE}	50
c	40 $	g	0.2
p	100$	k	0.2

Optimal results for all the three models are given in the table 3.

Table 3. Optimal Results of the three models

	t_2	T	Q	Total Profit

Model 4. 1	25.0393	30	68978.3	56706.0
Model 4. 2	25.330	30	70883.8	58415.57
Model 4. 3	25.493	30	71978.8	59342.81

6. Concavity

The concavity for all the three models are given by figure.1 , figure 2 and figure 3

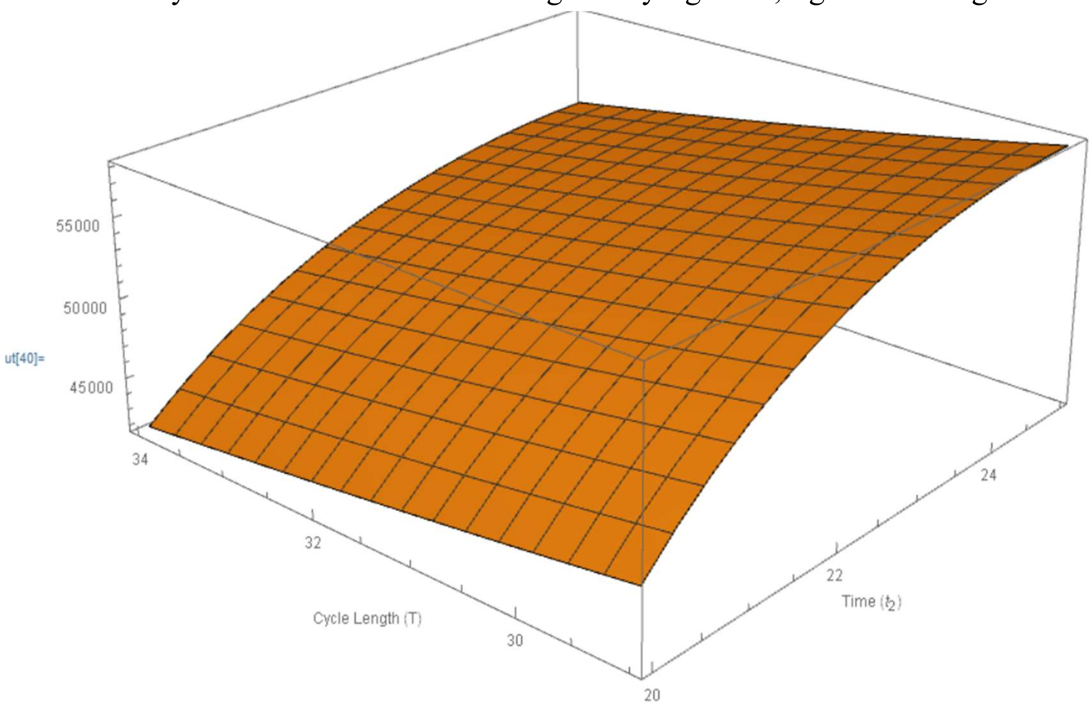

Figure 1. Concavity for model 4.1

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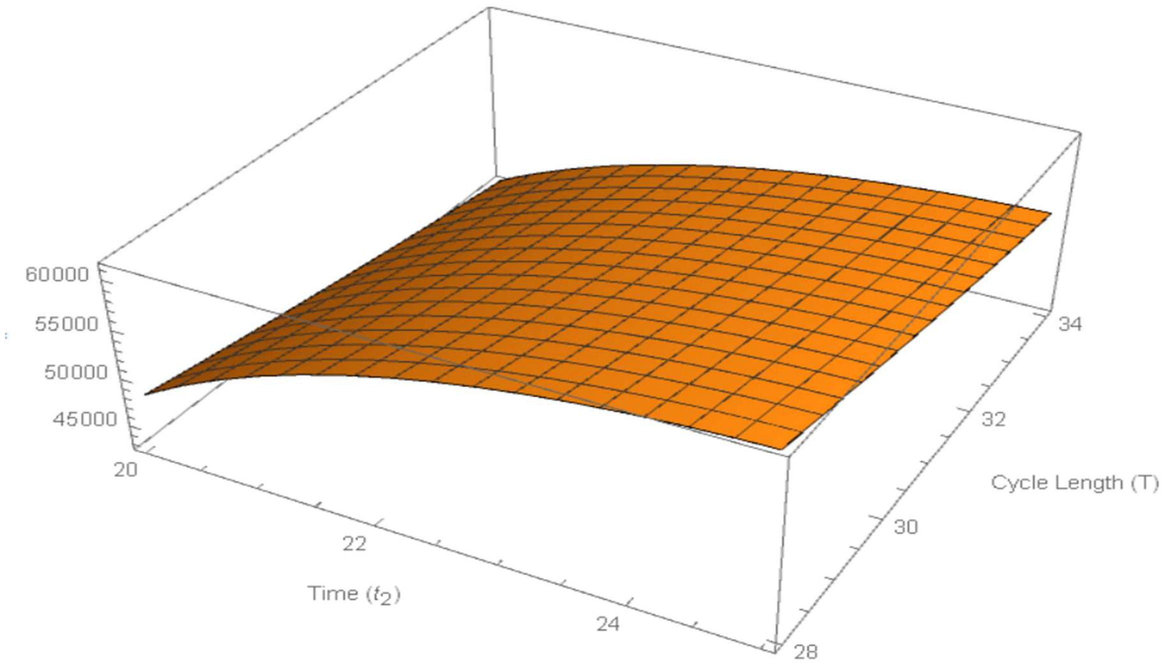

Figure 2. Concavity for model 4.2

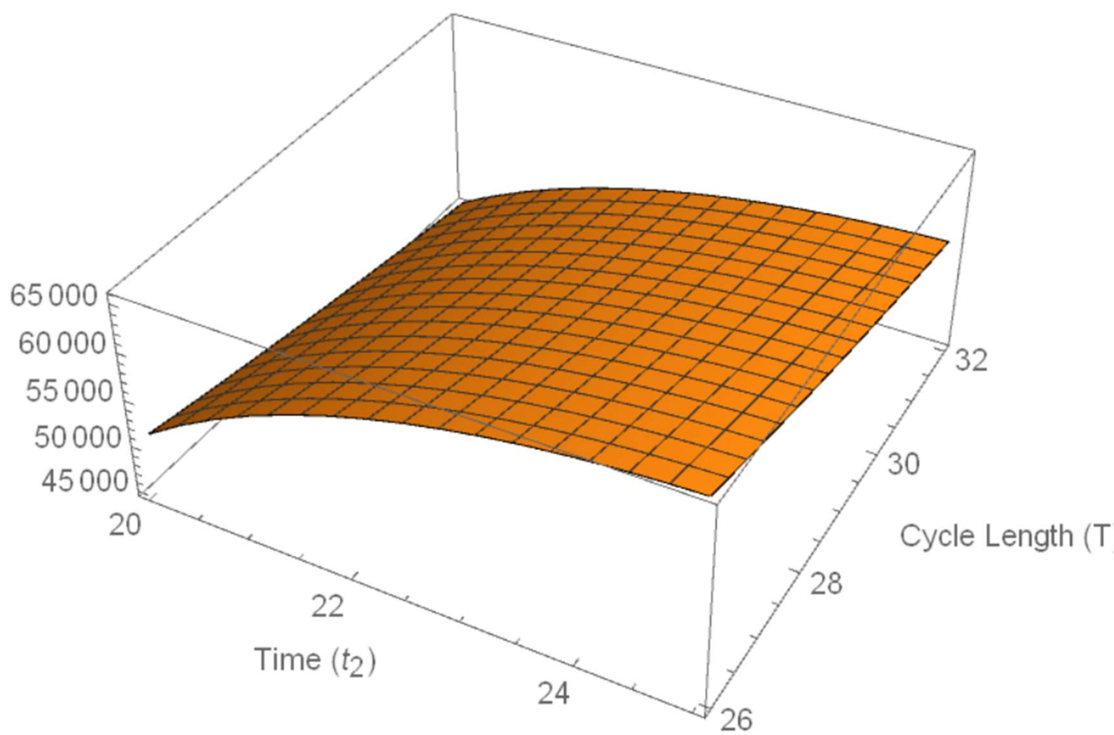

Figure 3. Concavity for model 4.3

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7. Sensitivity Analysis

Table 4. Sensitivity Anaalysis of various parameters in all the three models.

Parameters	% Change	Model with Carbon Tax Policy without Green Technology		Model with Carbon Tax Policy with Green Technology		Model with Carbon Cap and Trade Policy with Green Technology	
		t_2	TP_1	t_2	TP_2	t_2	TP_3
c	+20%	24.126	44792.1	24.3345	46314.9	24.4664	47149.8
	+10%	24.5504	50677.1	24.8016	52273.1	24.9422	53149.6
	-10%	25.6064	62909.6	25.9453	64703	26.1337	65692.8
	-20%	26.2674	69328.5	26.6634	71257.1	26.886	72325.2
p	+20%	26.3632	83258.7	26.6999	85199.8	26.8923	86270.3
	+10%	25.6881	69850.6	26.0098	71657.9	26.1894	72653.8
	-10%	24.4234	43814.8	24.68	45389.8	24.8243	46255.5
	-20%	23.8537	31161.1	24.0753	32640.5	24.2	33452.3
D	+20%	25.0393	68047.8	25.3303	70074.2	25.4935	71186.2
	+10%	25.0393	62376.9	25.3303	52548.7	25.4935	65252
	-10%	25.0393	51035.1	25.3303	46706.9	25.4935	53383.6
	-20%	25.0393	45364.2	25.3303	42324.0	25.4935	47449.4
β	+20%	24.7262	56089.6	25.003	57717.7	25.1543	58613.2
	+10%	24.8816	56390	25.1642	58045.9	25.3229	58957
	-10%	25.199	57037.9	25.4981	58752	25.6658	59695.8
	-20%	25.3606	57385.9	25.6676	59130.3	25.8394	60091
A	+20%	25.0393	56705.5	25.3019	58080.4	25.4935	59317.3
	+10%	25.0393	56705.8	25.3161	58235.4	25.4935	59317.6
	-10%	25.0393	56706.2	25.3445	58545.8	25.4935	59318.0
	-20%	25.0393	56706.5	25.3588	58701.2	25.4935	59318.3
r	+20%	24.5702	59459.7	24.9247	60801.5	25.117	61546.6
	+10%	24.4235	58594.6	24.7936	60035.9	24.9955	60836.5
	-10%	28.4647	46141.9	28.6338	488811	28.7283	50255.7
	-20%	34.8446	17700	34.8786	17675	34.9516	17675
h	+20%	24.8994	55776.4	25.1788	57435.2	25.3355	58347.6
	+10%	24.9686	56239.6	25.2538	57911.1	25.4137	58830.9
	-10%	25.1113	57175.6	25.4083	58873.6	25.575	59808.6
	-20%	25.1849	57648.4	25.488	59360.3	25.6582	60303.2
g	+20%	25.1004	56172.0	25.3767	57866.2	25.5319	58797.6
	+10%	25.0707	56438.7	25.3542	58128.2	25.5135	59057.6
	-10%	25.0057	56973.9	25.3049	58653.3	25.4726	59578.3
	-20%	24.9698	57242.6	25.2779	58916.4	25.4504	59839

θ	+20%	23.2546	52410.7	23.4616	53993.9	23.5774	54862.6
	+10%	24.0528	54467.3	24.2951	56092.3	24.431	56935.2
	-10%	26.3045	59200.3	26.6623	60971.8	26.8626	61949
	-20%	27.9919	62079.1	28.4378	63980	28.6858	65031
G	+20%	----	----	25.3356	58410	25.4963	59322.3
	+10%	------	----	25. 3336	58404.2	25.4952	59322.2
	-10%	-----	----	25.3248	58364.1	25.4907	59306.7
	-20%	-----	----	25.3159	58317	25.486	59284.8

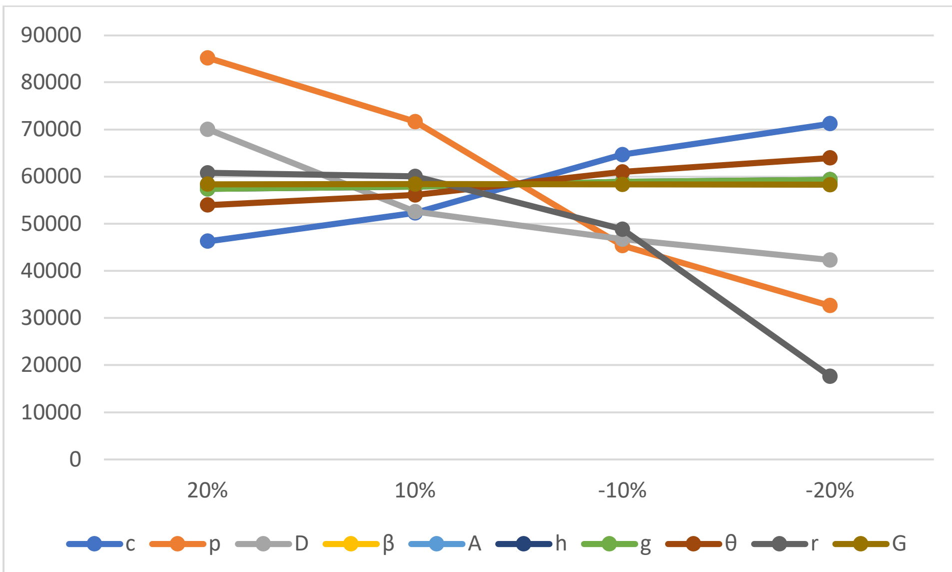

Figure 4. Effect of different parameters on total probit in model 4.1

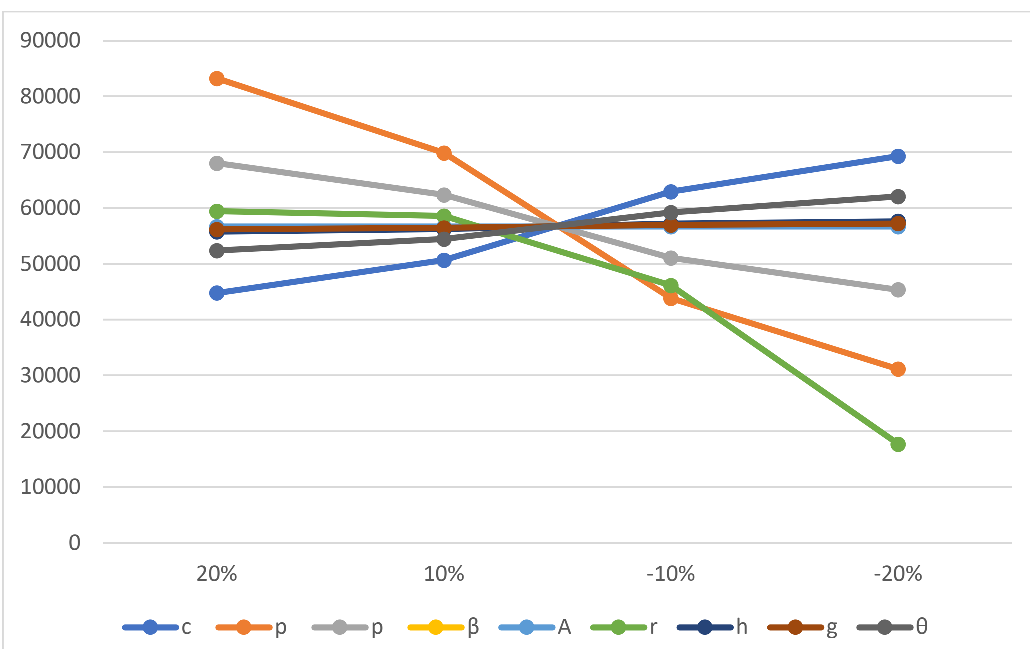

Figure 5. Effect of different parameters on total probit in model 4.2

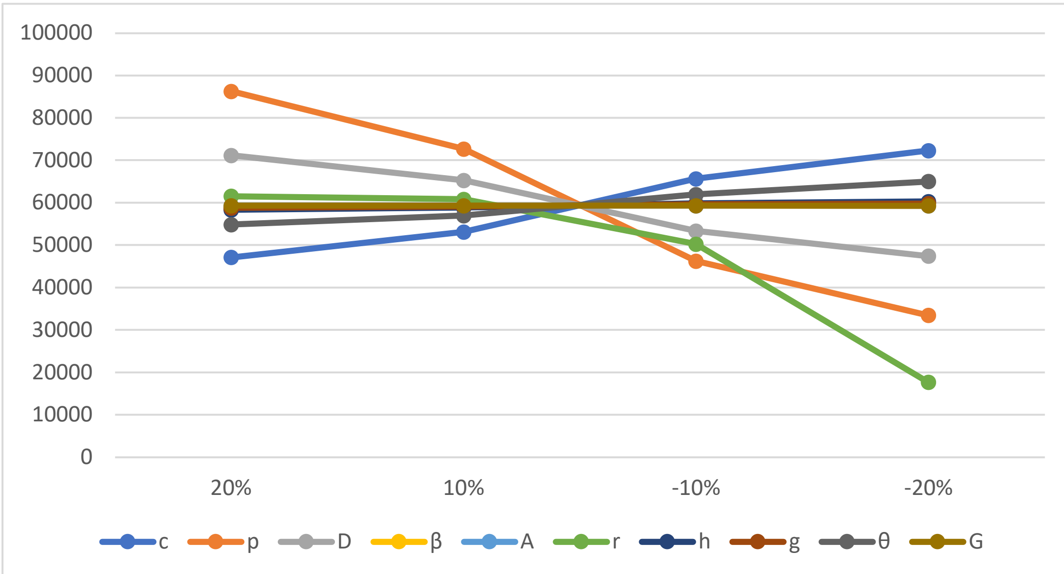

Figure 6. Effect of different parameters on total probit in model 4.3

8. Observations

i. On increases in the purchasing cost the total profit and cycle length decreases. Thus lower the purchasing price higher will be profit in all three models.

ii. On increases in the selling price the total profit and cycle length increases in all three models. So, the selling price positively affected the total profit in all three models.

iii. Demand rate positively impact on total profit. That is higher the demand higher will be profit in all three models.

iv. On increases in the backlogging rate the total profit and cycle length decreases in all three models.

v. On increases in the Order quantity the total profit is slightly decreases in all three models.

vi. On increases in the inflation rate the cycle length decreases while total profit increases in all three models.

vii. On increasing in the holding cost parameters ‘h’ and ‘g’ total profit in all three models. Decreases while cycle time decreases when h increases and cycle time increases when ‘g’ increases in all three models.

viii. On increases in the deterioration rate the total profit and cycle length decreases in all three models.

ix. On increases in the investment in green technology in model 4.2 and model 4.3 the total profit increases.

9. Conclusion

In this paper we developed sustainable inventory model in three different situations. First model is developed under carbon tax policy without investment in green technology. Second model is developed under carbon tax policy with investment in green technology. Third model is developed under carbon cap and trade policy with investment in green technology. Demand is taken as constant. Deterioration is non-instantaneous. Holding cost is variable. Effect of inflation is taken into account. Carbon emission is considered from ordering, holding and purchasing. Numerical illustration is carried out by using the software Mathematica 12.0. From the numerical results we conclude that model with carbon cap and trade policy with investment in in green technology have more profit. Sensitivity analysis is carried out to show the effect of different parameters on optimal results. From the sensitivity analysis we analysed that the lower the deterioration rate higher will be profit. So, manager should try to reduce the deterioration rate. Also, manager should incorporate the effect of inflation which results the higher profit. Investment in green technology also results the higher profit. This model can be extended with stochastic demand, under fuzzy environment and with different carbon emission policies.

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