China Petroleum Processing and Petrochemical Technology

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2503-2507

One-Way Tape Determinsitic PDA

Dr. Rajesh Kumar

Assistant Professor, CRM Jat College, Hisar, Haryana, INDIA

EMail ID: rajtaya@kuk.ac.in

Keywords: Automata, Deterministic, Pushdown, Quotient, Transition Functions.

Abstract. An automata with a tape is known as a pushdown automata (PDA). A pushdown automation has an advantage of scanning the alphabets of its tape without deleting its content. A one-way tape deterministic pushdown automata ($DPDA$) is considered in this paper. In this paper, it is presented that L(R) ={w| $x \in R, wx \in L\}$ where R is a set of symbols and a language L accepted by a PDA. On the basis of a corollary, the terminal symbols are not required on the alphabet set of $DPDA$. In this paper it is also presented that $Max_{(L)}$ = {w| $x \in R, wx \in L\}$ is accepted by a $DPDA$ only and only if L is accepted by a $DPDA$.

Introduction

A $DPDA$ has been studied as an acceptance device [1, 2]. The basic nature of PDA is nondeterministic consisting of two-way tape with terminal symbols and a transition function [3, 4]. The behavior of proposed DPDA is totally different form the existing PDA [5], the DPDA [6] searches the tape in a read-only manner. In the theoretical computer science, the closure properties on the existing language family has proven important. Some closure properties of deterministic PDA have been well-established for novel $DPDA$ [7, 8]. The following operations have been shown closed for the languages accepted by the proposed novel $DPDA$.

- Kleen Closure
- Quotient with a Regular Set
- Union
- Concatenation
- Intersection

In this paper, author try to establish analogous result for the languages for novel $DPDA$ which was previously unknown. The more interest in this result originate from the fact that all the existing axioms except complement, have been shown closure for proposed $DPDA$ [1, 2]. Furthermore, there exist some families of languages accepted by novel $DPDA$ that are not closed under quotient [5]. The method used in this paper for proof is new and has some other applications. The elimination of terminal symbols is one more fact to increase the existing interest in quotient (on regular set) L_1/L_2= { {w| $x \in L_2, wx \in L_1\}$ where L_1 and L_2 are languages and L_1/L_2 is the quotient. For $DPDA$, it is admitted that terminal symbols on the alphabet are avoidable for the non-deterministic PDA [9].

Definition of DPDA

Formally, $DPDA$ represented as 1_{WPDA} is a seven-tuple $(Q_k, \Sigma, \delta_B, \delta, Q_o, Q_F, \eta)$ machine such that

1. "$Q_k :\ a\ finite\ set\ of\ states$"
2. "$\Sigma :\ a\ non{-}empty\ finite\ set\ of\ input\ symbols$"

3. *"δ : is a transition from $\delta(Q_k, \Sigma \cup \varepsilon, \eta - \{B\}) \subseteq \delta(Q_k, S, R, L\})$ where range of δ is that the tape head remains stationary, moves right or moves left respectively"*
4. *"δ_B : is a transition from $\delta(Q_k, \Sigma \cup \varepsilon, \eta - \{B\}) \subseteq \delta(Q_k, S, E, L\}) \cup (\eta - \{B\})$ when top of the tape has blank symbol and E imply a move left followed by writing blank over rightmost non − blank; L implies a move left without erasing rightmost non − blank respectively."*
5. *"Q_o : an initial state"*
6. *"Q_F : a finite set of final state where $Q_F \subseteq Q$"*
7. *"η : a finite set of tape symbols including B & Z_o where B is blank and Z_o is top of tape"*

Configuration of Transition Function δ of 1_{WPDA}

A δ has three arguments Q, $\Sigma \cup \varepsilon$ and $\eta - \{B\}$. When the second argument of δ is ε the tape head moves either static (S) or right (R) or left (L) without reading any symbol from the input. The movement of δ transition function also depends on the B_r (non-blank rightmost symbol) with Q_c (current state) and Σ (input symbol). The initial configuration of $DPDA$ is $\delta(Q_k, \Sigma \cup \varepsilon, \eta_B \cup Z_o)$. It has been noted that Z_o is never written by $DPDA$, Z_o is an indicator of left end of the tape. One important fact is presumed that the tape head never moves beyond Z_o to the left i.e δ $(Q_p, \Sigma \cup \varepsilon, Z_o)$ does not contain (Q_q, L).
The initial configuration of transition function of 1_{WPDA} is denoted by δ $(Q_i, \varepsilon_y, \eta_i)$, where $Q_i \subset Q$, $\varepsilon_y \subset \Sigma \cup \varepsilon$ and η_i is an integer having value either 0 or 1. The value of η_i=0 represents that the position of head is on the top of tape and the rightmost non-blank symbol is represented by η_i=1. In this paper the configuration of transition function is defined as under:

1. *$\eta_i > 0$, if $(Q_1, D) \in \delta(Q_k, y, Z_0)$ | $y : (Q_k, x_1Z_0x_2, \eta_i) \vdash_{1_{WPDA}} (Q_1, x_1Z_0x_2, \eta_i + \gamma)$ where γ = -1, 0 or +1 respectively as $D = R, S$ or L.*
2. *if $(Q_1, L) \in \delta_B(Q_k, y, Z_0)$ | $y : (Q_k, xZ_0, 0) \vdash_{1_{WPDA}} (Q_1, xZ_0, 1)$.*
3. *if $(Q_1, E) \in \delta_B(Q_k, y, Z_0)$ | $y : (Q_k, xZ_0, 0) \vdash_{1_{WPDA}} (Q_1, x, 0)$.*
4. *if $(Q_1, S) \in \delta_B(Q_k, y, Z_0)$ | $y : (Q_k, xZ_0, 0) \vdash_{1_{WPDA}} (Q_1, xZ_0, 0)$.*
5. *if $(Q_1, Z_i) \in \delta_B(Q_k, y, Z_0)$ | $Z_i \in \eta$ & $y : (Q_k, xZ_0, 0) \vdash_{1_{WPDA}} (Q_1, xZ_0Z_i, 0)$.*
   If for $1 \leq j \leq n$, $y_j : (Q_j, x_j, \eta_j) \vdash_{1_{WPDA}} (Q_{j+1}, x_{j+1}, \eta_{j+1})$, then
   | $y_1y_2.....y_n : (Q_1, x_1, \eta_1) \vdash *_{1_{WPDA}} (Q_{n+1}, x_{n+1}, \eta_{n+1})$.

So, on the basis of above description it is stated that any automata, to be a (1_{WPDA}) if, intuitively from the above given configuration there is at most one move possible. A language (L) is always accepted by a final state of the given $DPDA$. Let the language L(1_{WPDA}) accepted by a 1_{WPDA}, is the set w | w : $(Q_0, \eta_0, 0) \vdash *_{1_{WPDA}} (Q_k$, x, $\eta_i)$ for some $Q_k \in Q_F$, x $\in \eta$ and η_i for some integer i) is called a 1_{WPDA} language.

Finding

With the help of lemma the authors try to simplify proof of main theorem in this section.
Theorem 1. Let A be a $DPDA$ and assume $X_1 = (Q_1, Z_1), (Q_2, Z_2), \ldots, (Q_m, Z_m)$ and $X_2 = (R_1, Y_1), (R_2, Y_2), \ldots, (R_n, Y_n)$ are sets of state instance and tape symbol.
Proof: Let $w \subseteq$ L be a language where $w \in I^*$ such that either

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2503-2507

1. $w : (Q_0, Z_0, 0) \vdash *_{1_{WPDA}} (Q_k, xZ, 0)$ for some x in η-$\{B\}^*$ and (Q_k, Z_0) in x_1, or

2. $w : (Q_0, Z_0, 0) \vdash *_{1_{WPDA}} (Q_k, x_1 Z x_2, j)$ where $j = |x_2| + 1$ and (Q_k, Z_0) in x_1.

In the above given description, authors assume $1'_{WPDA}$ is a newly constructed $DPDA$ from 1_{WPDA} such that $1'_{WPDA}$ accept the language L'. The formal definition of this newly $DPDA$ is as under: $1'_{WPDA} = (Q'_k, \Sigma', \delta'_B, \delta', Q'_o, Q'_F, \eta')$ where $Q' = \{Q_0, Q'_0, Q''_0 \mid Q_0 \in Q_k\}$, $Q_F = Q' \mid Q' \in Q_k$ and δ'_B, δ' are defined as under.

1. If $\delta(Q_k, x, \eta) = (Q_1, D)$ where $D \in \{L, S, R\}$ then $\delta'(Q_k, x, \eta) = (Q''_1, D)$.

2. If $(Q_k, \eta) \in x_2$, then $\delta'(Q'', \epsilon, \eta) = (Q', S)$ and $\delta'(Q', \epsilon, \eta) = (Q_k, S)$.

3. If $(Q_k, \eta) \notin x_2$, then $\delta'(Q'', \epsilon, \eta) = (Q_k, S)$.

4. If $\delta(Q_k, x, \eta) = (Q_1, D)$ where $D \in \{S, L, E\} \cup \Sigma - \{\eta - Z_0\}$ then $\delta'_B(Q_k, x, \eta) = (Q''_1, D)$.

5. If $(Q_k, \eta) \in x_1$, then $\delta'_B(Q'', \epsilon, \eta) = (Q', S)$ and $\delta'_B(Q', \epsilon, \eta) = (Q_k, S)$.

6. If $(Q_k, \eta) \notin x_1$, then $\delta'_B(Q'', \epsilon, \eta) = (Q_k, S)$.

The behaviour of transition functions δ and δ_B clearly indicate that language $L(1'_{DA})$ accepted by novel $DPDA$ is regular language including the empty tape.

Theorem 2. Let S is a finite state machine and 1_{WPDA} be a $DPDA$ so $\exists$ a $1'_{DA} = (Q_k, \Sigma, \delta'_B, \delta', Q_o, Q_F, \eta \times C, [Z_0 \cup \varnothing_o$ (mapping)] such that

$x : (Q_0, [Z_0 \cup \varnothing_o], 0) \vdash *_{1_{WPDA}} (Q_k, [Z_0 \cup \varnothing_o][Z_1 \cup \varnothing_1] \ldots [Z_n \cup \varnothing_n], i)$

iff

$x : (Q_0, [Z_0 \cup \varnothing_o], 0) \vdash *_{1_{WPDA}} (Q_k, Z_0 Z_1 \ldots Z_n, i)$, and for $0 \le i \le n$, $_i$ describe the mapping i.e $Z_0 Z_1 \ldots Z_i$.

Proof: The behaviour of transition functions δ' and δ'_B are defined as under.

1. $\delta'(Q, \alpha, [\eta, \varnothing]) = \delta(Q, \alpha, \eta)$. $1'_{DA}$ moves exactly as 1_{WPDA} if the tape head of $1'_{DA}$ is in the tape and $1'_{DA}$ ignore the mapping.

2. $\delta_B(Q, \alpha, Z_1) = (Q_1, Z_2)$ where $Z_2 \in \eta$, then $\delta'_B(Q, \alpha, [\eta, \varnothing_1]) = (Q_1, [Z_2, \varnothing_2])$ such that yZ_2 described by the mapping function $\varnothing_2$ where $y \in \eta*$.

These above transition function move clearly satisfies the theorem proof.

Theorem 3. Let 1_{WPDA} be a $DPDA$ and $1'_{WPDA}$ be a newly constructed machine from 1_{WPDA} according to theorem 2. So, $\exists$ a $1''_{DA} = (Q'', \Sigma, \delta''_B, \delta', Q_o, Q_F, \eta \times C, [Z_0 \cup \varnothing_o])$ such that

1. $x : (Q_0, [Z_0 \cup \varnothing_o], 0) \vdash^*_{1''_{DA}} (Q_k, [Z_0, \varnothing_o][Z_1, \varnothing_1] \ldots [Z_n, \varnothing_n], 0)$
   $iff \quad x : (Q_0, [Z_0 \cup \varnothing_o], 0) \vdash *_{1_{WPDA}} (Q_k, Z_0 Z_1 \ldots Z_n, 0)$

2. $x : (Q_0, [Z_0 \cup \varnothing_o], 0) \vdash^*_{1''_{DA}} ([Q_k, m_i], [Z_0, \varnothing_o] \ldots [Z_n, \varnothing_n], n - i + 1)$, $i > 0$
   $iff\ x^{i=n}_{i=0} : \vdash *_{1_{WPDA}} (Q_k, Z_0 Z_1 \ldots Z_n, 0)$ where $[Z_0, m_i]$ is associated with the mapping function i.e $[Z_i, \varnothing_i]$

$Proof$: The behaviour of the transition functions of $DPDA$'s $1''_D A$ and $1_D A'$ look like similar except the tape movement of $1'_{WPDA}$. So,

1. if $\delta'_B(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, S)$, (Q_1, E) or $(Q_1, [Z_1, \varnothing_1])$ then
   $\delta''_B(Q_k, \alpha, [\eta, \varnothing]) = \delta'_B(Q_k, \alpha, [\eta, \varnothing])$

2. if $\delta'_B(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, \text{L})$ then $\delta''_B(Q_k, \alpha, [\eta, \varnothing]) = [z_1, m_0], \text{L})$
   where top of the tape point (B_r) non-blank symbol which is associated with the mapping function $m_0 \in [\eta, \varnothing]$.
3. For every Q in Q_k, let $\hat{Q} \in \hat{Q}_k$ where $\hat{Q}_k$ is a set of all new symbol $\hat{Q}$ such that
4. if $\delta'(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, \text{L})$, then each $m \in M$, $\delta''([Q_k, m], \alpha_1, [\eta, \varnothing]) = (\hat{Q}, \alpha, [\eta, \varnothing])$
5. if $\delta'_B(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, \text{L})$ then $\delta''_B(Q_k, \alpha, [\eta, \varnothing]) = [Z_1, m_0], \text{L})$
6. For every $\hat{Q} \in \hat{Q}_k$, $m \in M$ and $[\eta, \varnothing] \in \eta \times \text{C}$,
   $\delta''([\hat{Q}_1, m], \epsilon, [\eta, \varnothing]) = ([Q_1, \triangle\ (m_0), [\eta, \varnothing]], S)$
7. if $\delta'(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, S)$, then each $m \in M$, $\delta''([Q_k, m], \alpha, [\eta, \varnothing]) = ([Q_1, m], S)$

Somehow, if inverse of the $\triangle(m, [\eta, \varnothing])$ i.e $\triangle^{-1}(m, [\eta, \varnothing])$ may be considered and if it determines $[\eta, \varnothing]$ uniquely then it represents the transition $\delta'(Q_k, \alpha, [\eta, \varnothing]) = (Q_1, R)$ with the right symbol of $[\eta, \varnothing]$. If $\triangle^{-1}(m, [\eta, \varnothing]) = \{m_1, m_2, ...m_r\}$ where $r \geq 2$.

Main result

With the proof of $theorem$ 2 it is clearly visible that the class of language produced by 1_{WPDA} are closed under the $quotient\ with\ regular\ set$.
$Theorm$ 4. L/R is accepted by 1_{WPDA} if and only if $L \subseteq \Sigma \in \forall\ Q_F$ and $R \subseteq \Sigma$ is a regular set.
$Proof$: Let $\Psi \notin \epsilon$ and $R = R\Psi$. Let S be a $DPDA$ $S = (Q_S, \epsilon \cup \Psi, \delta_s, Q_0, Q_F)$ is accepting R'. On the basis of $Theorm$ 1, construct a $DPDA$ A which accepts $L\Psi$ by empty tape. By $Theorm$ 2 and $Theorm$ 3 construct $1''_{WPDA}$ from 1_{WPDA}. The two components X_1 and X_2 are defined as under:
X_1: $\{Q_k, [\eta, \varnothing]\}\ \exists\ Q_1 \in Q_k$ such that $\hat{\delta}(Q_0, P_0, Q_1, P_1)$
X_2: $\{[Q_k, m], [\eta, \varnothing]\}$, $m(Q_k, P_0 = 1))$

1. $x \mid x : (Q_0, [Z_0, \varnothing_o], 0) \vdash^*_{1''_{DA}} (Q_k, y[\eta, \varnothing], 0\ ), (Q_k, [\eta, \varnothing], 0\ ) \in X_1\ or$
2. $x \mid x : ([Q_0, m], [Z_0, \varnothing_o], 0) \vdash^*_{1''_{DA}} (Q_k, y_1[\eta,]y_2, j)$, $j = |y_2 + 1|$ and $([Q_k, m], [\eta, \varnothing]\ ) \in X_2$.[10]

Summary

Max is an operation defined by $Max_{(L)}$ which represents an operation strongly related to quotient with a regular set.
$Max_{(L)} = \{\alpha \mid \alpha \in L$ and $\beta \notin \Sigma^* - \{\varepsilon\}$ is $\alpha\beta$ in $L\ \}$. By modifying the transition functions of $Theorm$ 2 and $Theorm$ 3 it can be shown that the family of languages accepted by non-deterministic non-erasing PDA [1] is closed under quotient with a regular set. With the help of this alternate proof it can be clearly concluded that the family of languages accepted by 1_{WPDA} is properly contained in the family of languages accepted by an 1_{DPDA}.

References

[1] Aho, A. Computational thinking in programming language and compiler design (keynote). *STOC '21: 53rd Annual ACM SIGACT Symposium On Theory Of Computing, Virtual Event, Italy, June 21-25, 2021*. pp. 1 (2021), https://doi.org/10.1145/3406325.3465350

[2] Aho, A. & Ullman, J. Abstractions, their algorithms, and their compilers. *Commun. ACM*. **65**, 76-91 (2022), https://doi.org/10.1145/3490685

[3] Aho, A. Computation and Computational Thinking. *Comput. J.*. **55**, 832-835 (2012), https://doi.org/10.1093/comjnl/bxs074

[4] Okhotin, A. & Salomaa, K. Descriptional complexity of unambiguous input-driven pushdown automata. *Theoretical Computer Science*. **566** pp. 1-11 (2015), https://www.sciencedirect.com/science/article/pii/S0304397514008585

[5] Ullman, J. The Battle for Data Science. *IEEE Data Eng. Bull.*. **43**, 8-14 (2020), http://sites.computer.org/debull/A20june/p8.pdf

[6] Polách, R., Trávníček, J., Janoušek, J. & Melichar, B. Efficient determinization of visibly and height-deterministic pushdown automata. *Computer Languages, Systems & Structures*. **46** pp. 91-105 (2016), https://www.sciencedirect.com/science/article/pii/S1477842416300136

[7] Grigorian, H. & Shoukourian, S. The equivalence problem of multidimensional multitape automata. *J. Comput. Syst. Sci.*. **74**, 1131-1138 (2008), https://doi.org/10.1016/j.jcss.2008.02.006

[8] Kutrib, M. & Malcher, A. Context-dependent nondeterminism for pushdown automata. *Theoretical Computer Science*. **376**, 101-111 (2007), https://www.sciencedirect.com/science/article/pii/S0304397507000321, Developments in Language Theory

[9] Ullman, J. On the Nature of Data Science. *KDD '21: The 27th ACM SIGKDD Conference On Knowledge Discovery And Data Mining, Virtual Event, Singapore, August 14-18, 2021*. pp. 4 (2021), https://doi.org/10.1145/3447548.3469651

[10] Fernau, H., Wolf, P. & Yamakami, T. Synchronizing Deterministic Push-Down Automata Can Be Really Hard. *45th International Symposium On Mathematical Foundations Of CS, MaFCS 2020, August 24-28, 2020, Prague, Czech Republic*. **170** pp. 33:1-33:15