China Petroleum Processing and Petrochemical Technology

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2396-2410

THEORETICAL STUDY ON COSMOS USING ALTERNATIVES THEORIES OF GRAVITATION

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Abstract: In this paper we studied a Five dimensional FRW space-time model in the presence of bulk viscous fluid in the frame work of Brans and Dicke (Phys. Rev. 124:925, 1961) scalar–tensor theory of gravitation. At the same time the we studied another model known as spatially homogeneous and anisotropic Kantowski-Sachs space-time is considered in the scale covariant theory of gravitation proposed by Canuto et al. (Phys. Rev. Lett.39:429,1977) . We obtain determinate solution of the field equations. Physical properties of the models are also discussed.
Key words: *Scale covariant theory, Bulk viscous model, String model, Kantowski-Sachs model, Brans–Dicke theory, FRW models*

1. Introduction:

It is well known that matter distribution is satisfactorily described by a perfect fluid due to a large scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the condition of material distribution other than the perfect fluid. We know that when neutrino decoupling occurs, the matter behaves as a viscous fluid in an early stages of the universe [16]. Bulk viscosity plays an important role in cosmology in getting the recent scenario of accelerated expansion of the universe [19, 20] popularly known as the inflationary phase. Hence in recent years cosmological models with bulk viscosity has become an important subject of investigation. Both in general relativity and in modified theories of gravitation [17]. It was shown that inflationary model [18], extended inflationary model [13], hyper extended inflationary model [24], Chaotic inflation [14] are based BD theory. In this theory, besides a gravitational part, a dynamical scalar field has been introduced to account for the variable gravitational constant to incorporate Mach's principle.

Study of five dimensional space time is important because of the fact that cosmos at its early stage of evolution might have had a higher dimensional era. Marciano[15] suggested that the experimental detection of time variation of fundamental constants provide strong evidence for the existence of extra dimension. The extra dimension in the space time contracted to a very small size of Planck length or remain invariant. Further, during contraction process extra dimensions produce large amount of entropy which provides an alternative resolution to the flatness and horizon problem [1,10] were attracted to the study of higher dimensional cosmology because it has physical relevance to the early times before the universe has undergone compactification transitions.

In formulating general theory of relativity, Einstein was guided by principle of covariance, principle of equivalence and principle of Mach. However, Einstein himself pointed that Einstein's theory of gravitation does not incorporate Mach's principle satisfactorily. Hence several

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2396-2410

alternative theories of gravitation have been formulated from time to time. Also, the recent discovery of accelerated expansion of the universe[19,20] stimulated interest in alternative theories of gravitation. Noteworthy among them are scalar-tensor theories of gravitation formulated by Brans and Dicke [6] and Saez and Ballester [21], modified theories like f(R) [8] and f(R,T) [11] theories of gravity. Brans-Dicke (BD) theory introduces an additional scalar field Ø interacting equally with all forms of matter besides the metric tensor g_{ij} and dimensionless coupling constant ω. This scalar field is introduced to account for variable gravitational constant. Saez and Ballester[21] introduced a scalar-tensor theory of gravity in which metric is coupled to gravitational constant. This theory helps to solve the 'missing mass' problem. It is well known that theories of gravity with scalar fields are most important due to their vast cosmological implications [5]. Scale covariant theory of gravitation proposed by Canuto et al.[7] is another important modification of Einstein's theory of gravitation. This theory is a viable alternative general relativity [25,26] which also admits a variable gravitational 'constant'. In this theory physical quantities are measured in atomic units whereas Einstein's field equations are valid in gravitational units.

In view of the importance and relevance of modified theories of gravitation to modern cosmology, investigation of cosmological models in these theories are attracting more and more attention. Cosmic strings and Bulk viscosity play a vital role in the early stages of evolution of the universe. Strings arise as a random network of stable line-like topological defects during the phase transition in the early universe. Massive closed loops of strings serve as seeds for the formation of large structures like galaxies and cluster of galaxies at the early stages of evolution of the universe. Bulk viscosity contributes negative pressure term giving rise to an effective total negative pressure stimulating repulsive gravity which overcomes attractive gravity of matter and gives an impetus to the accelerated expansion of the universe popularly known as the inflationary phase. In view of the above discussion, bulk viscous cosmic string models in modified theories of gravitation have been investigation by several authors. In particular, The above discussion and the investigations inspired us to investigate, in this paper, Kantowski-Sachs bulk viscous cosmic string cosmological models in the scale covariant theory of gravitation proposed by Canuto et al.[7].The present article is organized as follows: In Sect. 2 & Sec.3 presented two cases which includes the field equations and physical parameters of the models have been derived. Section 4 deals with the some conclusions about the models.

2. <u>Case-1</u>

2.1 Metric and field equations

We consider FRW five dimensional space time metric in the form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\emptyset^2) + (1-kr^2)d\Psi^2\right] \tag{1}$$

where a(t) is the scale factor of the universe, $k = 1,\ 0,\ -1$ for space of positive, vanishing and negative curvature representing closed, flat and open models of the universe respectively. The non-zero components of the Einstein tensor G_j^i for (3.1) are given by

$$G_0^0 = G_1^1 = G_2^2 = G_3^3 = -\left(\frac{3(\dot{a})^2}{a^2} + \frac{3\ddot{a}}{a} + \frac{3k}{a^2}\right), G_4^4 = -\left(\frac{6(\dot{a})^2}{a^2} + \frac{6k}{a^2}\right) \tag{2}$$

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2396-2410

Now using commoving coordinates, Brans-Dicke field equations for the metric (1) can be derived as

$$\frac{6\dot{a}^2}{a^2}+\frac{6k}{a^2}+\frac{\omega}{2}\left(\frac{\dot{\emptyset}}{\emptyset}\right)^2+\frac{4\dot{a}\dot{\emptyset}}{a\emptyset}=8\pi\emptyset\rho \tag{3}$$

$$\frac{3\ddot{a}}{a}+\frac{3\dot{a}^2}{a^2}+\frac{3k}{a^2}+\frac{\omega}{2}\left(\frac{\dot{\emptyset}}{\emptyset}\right)^2-\frac{3\dot{a}\dot{\emptyset}}{a\emptyset}+\frac{\ddot{\emptyset}}{\emptyset}=8\pi\emptyset^{-1}\bar{p} \tag{4}$$

$$\ddot{\emptyset}+\frac{4\dot{a}\dot{\emptyset}}{a}=8\pi(3+2\omega)^{-1}(\rho-4\bar{p}) \tag{5}$$

$$\dot{\rho}+4\frac{\dot{a}}{a}(\rho+\bar{p})=0 \tag{6}$$

(For detailed derivation of above equations see **Appendix-I**)

The physical parameters $'H'$ & $'q'$ are defined by

$$H=\frac{\dot{a}}{a} \tag{7}$$

$$q=\frac{-(\dot{H}+H^2)}{H^2} \tag{8}$$

2.2 Solutions and the model

Field equations (3) to (6) are three independent equations in five unknowns namely $a,\emptyset,p,\rho$ & ζ [Equation (6) being the consequence of Equations (3) to (5)]. Hence we use the condition (2.7) and the well accepted relation between scalar field $\emptyset$ and the universe scale factor $a(t)$.

$$\emptyset=\emptyset_0 a^l \quad , \quad \text{where } \emptyset_0 \text{ and } l>0 \text{ are constants.} \tag{9}$$

$$T=T_i^i=\rho-4\bar{p}=0,\ \ i=0,1,2,3,4 \tag{10}$$

Closed model (k=1)

Here the field equations (3) to (5) with the support of equation (9) produce the succeeding solutions for the scale factor

$$a(t)=\left[\left(\frac{l+4}{l\emptyset_0}\right)(a_0t+t_0)\right]^{\frac{1}{l+4}} \tag{11}$$

Now with the proper choice of coordinates and constants, we can write the metric (1), with the help of (11), as (ie we choose $a_0=1, t_0=0$)

$$ds^2=\ dt^2-\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{2}{l+4}}\left[\frac{dr^2}{1-r^2}+r^2(d\theta^2+sin^2\theta d\emptyset^2)+(1-r^2)d\Psi^2\right] \tag{12}$$

Also the physical quantities are

$$\emptyset=\emptyset_0\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{l}{l+4}} \tag{13}$$

The model (12) signifies five dimensional FRW bulk viscous radiating models with the following physical parameters which are significant in the discussion of cosmos.

$$V=a^4=\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{4}{l+4}} \tag{14}$$

$$H=\left(\frac{1}{l+4}\right)\frac{1}{t} \tag{15}$$

$$8\pi\rho=\emptyset_0\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{l}{l+4}}\left[\frac{\omega l^2+8l+12}{2(l+4)^2t^2}+6\left\{\left(\frac{l+4}{l\emptyset_0}\right)t\right\}^{\frac{-2}{l+4}}\right] \tag{16}$$

$$8\pi p = \varepsilon_0 \emptyset_0 \left[\frac{\omega l^2+8l+12}{2(l+4)^2 t^2} + 6\left\{\left(\frac{l+4}{l\emptyset_0}\right)t\right\}^{\frac{-2}{l+4}}\right]\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{2}{l+4}} \quad (17)$$

$$8\pi\zeta = \emptyset_0\left(\frac{l+4}{3}\right)(\varepsilon-\varepsilon_0)\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{l}{l+4}}\left[\frac{\omega l^2+8l+12}{2(l+4)^2t^2} + 6\left\{\left(\frac{l+4}{l\emptyset_0}\right)t\right\}^{\frac{-2}{l+4}} t\right] \quad (18)$$

Open model (k = -1)

In this specific case, the model and the physical and kinematical quantities are

$$ds^2 = dt^2 - \left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{2}{l+4}}\left[\frac{dr^2}{1+r^2} + r^2(d\theta^2 + sin^2\theta d\emptyset^2) + (1+r^2)d\Psi^2\right] \quad (19)$$

$$8\pi\rho = \emptyset_0\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{l}{l+4}}\left[\frac{\omega l^2+8l+1}{2(l+4)^2t^2} - 6\left\{\left(\frac{l+4}{l\emptyset_0}\right)t\right\}^{\frac{-2}{l+4}}\right] \quad (20)$$

$$8\pi p = \varepsilon_0\emptyset_0\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{2}{l+4}}\left[\frac{\omega l^2+8l+12}{2(l+4)^2t^2} - 6\left\{\left(\frac{l+4}{l\emptyset_0}\right)t\right\}^{\frac{-2}{l+4}}\right] \quad (21)$$

$$8\pi\zeta = \emptyset_0\left(\frac{l+4}{3}\right)(\varepsilon-\varepsilon_0)\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{l}{l+4}}\left[\frac{\omega l^2+8l+1}{2(l+4)^2t^2} - 6\left\{\left(\frac{l+4}{l\emptyset_0}\right)t\right\}^{\frac{-2}{l+4}} t\right] \quad (22)$$

Flat model (k = 0)

In this particular case, the model and the physical and kinematical quantities are

$$ds^2 = dt^2 - \left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{2}{l+4}}[dr^2 + r^2(d\theta^2 + sin^2\theta d\emptyset^2) + d\Psi^2] \quad (23)$$

$$8\pi\rho = \emptyset_0\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{l}{l+4}}\left[\frac{\omega l^2+8l+12}{2(l+4)^2t^2}\right] \quad (24)$$

$$8\pi p = \varepsilon_0\emptyset_0\left[\left(\frac{l+4}{l\emptyset_0}\right)(t)\right]^{\frac{2}{l+4}}\left[\frac{\omega l^2+8l+12}{2(l+4)^2t^2}\right] \quad (25)$$

$$8\pi\zeta = \emptyset_0\left(\frac{l+4}{3}\right)(\varepsilon-\varepsilon_0)\left[\left(\frac{l+4}{l\emptyset_0}\right)t\right]^{\frac{l}{l+4}}\left[\frac{\omega l^2+8l+12}{2(l+4)^2t^2} t\right] \quad (26)$$

2.3 Physical discussion of the model

Equations (12), (19) and (23) represent FRW five dimensional radiating closed, open and flat models in BD theory .It might be apparent that there is no initial singularity for all the models. In closed and open models the p, ρ *and* ζ diverge initially and decrease with time. In the flat model the ρ, p *and* ζ decreasing with time and will become zero for enormously larger values of t. Also they all diverge at the initial epoch. The spatial volume in all the models is equal and increases with time and tends to infinity for substantially large time. The middling Hubble's parameter given by Equation (15) is similar for all the models and will diverge at the initial epoch, ie.at t=0 and will approach infinity as t becomes infinitely large. The models will support us to comprehend the spatially homogeneous and isotropic bulk viscous universe in five dimensions just before compactification transition. The scalar field in all the models increases with time the deceleration parameter in each case is $q = l + 3$ which shows that the models in five dimensions decelerate in the standard way.

3. Case-2

3.1 Metric and Field equations

Consider the Kantowski-Sachs space-time specified by

$$ds^2 = dt^2 - E^2dr^2 - F^2(d\theta^2 + Sin^2\theta\, d\varphi^2) \tag{27}$$

Kantowski-Sachs spacetimes (1966) signify anisotropic and homogeneous increasing (contracting) cosmologies. They also deliver models where the things of anisotropy can be projected and paralleled with FRW cosmologies. Also these space-times play a significant role in understanding the accurate picture of the universe immediately after the big bang.

Here the energy momentum tensor T_{ij} is specified by

$$T_{ij} = (\rho + \bar{p})\, u_i u_j - \bar{p} g_{ij} - \lambda x_i x_j \tag{28}$$

and $$\bar{p} = p - 3\zeta H \tag{29}$$

also, $u^i = \delta_4^i$, a four-velocity vector which fulfills

$$u^i u_j = 1,\ x^i x_j = 1 \quad and \quad u^i x_i = 0 \tag{30}$$

and $$T_1^1 = \lambda - \bar{p}\,,\ T_2^2 = T_3^3 = -\bar{p}\,,\ T_4^4 = \rho \tag{31}$$

Here $\rho\,, \bar{p}\,,\ \lambda\ and\ \zeta$ are functions of cosmic time t only.

Using commoving coordinates and eqs.(2) to (5) the field equations for the metric (1) yield the following equations

$$2\frac{\ddot{F}}{F} + \frac{\dot{F}^2}{F^2} + \frac{1}{F^2} + \frac{\ddot{\emptyset}}{\emptyset} - \frac{\dot{\emptyset}^2}{\emptyset^2} - \frac{\dot{\emptyset}}{\emptyset}\left(\frac{\dot{E}}{E} - 2\frac{\dot{F}}{F}\right) = 8\pi G(\emptyset)(\bar{p} - \lambda) \tag{32}$$

$$\frac{\ddot{E}}{E} + \frac{\ddot{F}}{F} + \frac{\dot{E}}{E}\frac{\dot{F}}{F} + \frac{\ddot{\emptyset}}{\emptyset} - \frac{\dot{\emptyset}^2}{\emptyset^2} + \frac{\dot{E}}{E}\frac{\dot{\emptyset}}{\emptyset} = -8\pi G(\emptyset)\bar{p} \tag{33}$$

$$2\frac{\dot{E}}{E}\frac{\dot{F}}{F} + \frac{\dot{F}^2}{F^2} + \frac{1}{F^2} - \frac{\ddot{\emptyset}}{\emptyset} + 3\frac{\dot{\emptyset}^2}{\emptyset^2} + \frac{\dot{\emptyset}}{\emptyset}\left(\frac{\dot{E}}{E} + 2\frac{\dot{F}}{F}\right) = 8\pi G(\emptyset)\rho \tag{34}$$

(For detailed derivation of above equations see **Appendix-II**)

$$\dot{\rho} + (\rho + p)u_j^k k + \rho\frac{(G\dot{\emptyset})}{G\emptyset} + 3\bar{p}\frac{\dot{\emptyset}}{\emptyset} = 0 \tag{35}$$

For the metric (27) this takes the form

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{E}}{E} + 2\frac{\dot{F}}{F}\right) + \rho\left(\frac{\dot{G}}{G} + \frac{\dot{\emptyset}}{\emptyset}\right) + 3\bar{p}\frac{\dot{\emptyset}}{\emptyset} = 0 \tag{36}$$

For the metric (7.1), we define the following parameters

And "V" is assumed by

$$V = b^3 = EF^2 \tag{37}$$

where $b(t)$ is the scale factor of the universe.

The 'H' is specified by

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\left(\frac{\dot{E}}{E} + 2\frac{\dot{F}}{F}\right) \tag{38}$$

Where H_1, H_2, H_3 are maneuvering Hubble's strictures in x,y and z directions.

The mean anisotropy parameter A_h is defined as

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2\ ,\Delta H_i = H_i - H, i = 1,2,3 \tag{39}$$

The scalar spreading out θ and shear scalar are given by

$$\theta = u_{;k}^k = \frac{\dot{E}}{E} + 2\frac{\dot{F}}{F} \tag{40}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{E}^2}{E^2} + \frac{\dot{F}^2}{F^2} - 2\frac{\dot{E}\dot{F}}{EF}\right] \tag{41}$$

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2396-2410

3.2 Solutions and the model

The field equations (31) to (33) and (36) reduce to the following equations.

$$\frac{\ddot{F}}{F}-\frac{\ddot{E}}{E}+\frac{\dot{F}^2}{F^2}-\frac{\dot{E}\dot{F}}{EF}+\frac{1}{F^2}-2\frac{\dot{\emptyset}}{\emptyset}\left(\frac{\dot{E}}{E}-\frac{\dot{F}}{F}\right)=8\pi G(\emptyset)\lambda \tag{42}$$

$$2\frac{\dot{E}\dot{F}}{EF}+\frac{\dot{F}^2}{F^2}+\frac{1}{F^2}-\frac{\ddot{\emptyset}}{\emptyset}+3\frac{\dot{\emptyset}^2}{\emptyset^2}+\frac{\dot{\emptyset}}{\emptyset}\left(\frac{\dot{E}}{E}+2\frac{\dot{F}}{F}\right)=8\pi G(\emptyset)\rho \tag{43}$$

These are two self-governing equations in five unknowns $A, B, p, \rho\ and\ \lambda$. Hence forth to find a determinate resolution we have

$$E=F^l,\ \text{where } l\neq 1 \tag{44}$$

$$\overline{p}=p-3\zeta H=\varepsilon\rho \tag{45}$$

where $\epsilon=\epsilon_0-\beta\ \ (0\leq\epsilon_0\leq 1)\ and\ \ p=\epsilon_0\rho, \epsilon_0\ and\ \beta$ are constants.

$$q=-b\frac{\ddot{b}}{\dot{b}^2}=\text{constant} \tag{46}$$

which admits the solution for scale factor

$$b(t)=(c_1t+c_2)^{1/1+q} \tag{47}$$

Now from Equations (7.11), (7.18) and (7.21) we obtain the expressions for the metric coefficients as

$$E=(c_1t+c_2)^{\frac{3l}{(1+q)(l+2)}},\quad F=(c_1t+c_2)^{\frac{3}{(1+q)(l+2)}} \tag{48}$$

By a suitable choice of integration constants (i.e. $c_1=1\ and\ c_2=0$) we can write the space-time (27) in the form

$$ds^2=dt^2-t^{\frac{6l}{(1+q)(l+2)}}dr^2-t^{\frac{6}{(1+q)(l+2)}}(d\theta^2+sin^2\theta d\varphi^2) \tag{49}$$

3.3 Physical Discussion

Equation (49) signifies KS bulk viscous string astrophysical model in a scale covariant theory of gravitation. In this model, the kinematical and physical parameters of cosmology are the following:

$$V=t^{3/1+q} \tag{50}$$

$$\theta=\frac{3}{(1+q)t} \tag{51}$$

$$\sigma^2=\frac{3(l-1)^2}{(l+2)^2(1+q)^2t^2} \tag{52}$$

$$A_h=0 \tag{53}$$

$$H=\frac{1}{(1+q)t} \tag{54}$$

$$8\pi G(\emptyset)\rho=\frac{18(2l+1)+(l+2)^2(1+q)(5+2q)}{2(1+q)^2(l+2)^2t^2}+t^{-\frac{6}{(1+q)(l+2)}} \tag{55}$$

$$8\pi G(\emptyset)p=\sum\left[\frac{18(2l+1)+(l+2)^2(1+q)(5+2q)}{2(1+q)^2(l+2)^2t^2}+t^{-\frac{6}{(1+q)(l+2)}}\right] \tag{56}$$

$$8\pi G(\emptyset)\zeta=\left(\frac{\epsilon_0-\epsilon}{3}\right)(1+q)t\left[\frac{18(2l+1)+(l+2)^2(1+q)(5+2q)}{2(1+q)^2(l+2)^2t^2}+t^{-\frac{6}{(1+q)(l+2)}}\right] \tag{57}$$

The above results can be used for a physical discussion of the universe represented by equation (7.23).The model does not possess initial singularity i.e. at $t=0$.The universe exhibits spatial expansion as t increases because $1+q>0$, The parameters $\theta, \sigma, H, p, \rho, and\ \zeta$ diverge for $t=0$ and vanish as $t\rightarrow\infty$. It may be observed that the scalar field in the model vanishes at $t=0$

and becomes infinite as t increases indefinitely. It can be seen that $\frac{\sigma^2}{\theta}$ decreases with time which shows that anisotropy dies down with time and ultimately the model becomes isotropic in view of the fact that the average anisotropy parameter A_h in this model vanishes. This shows that there is a transition from deceleration to the accelerated phase of the universe at late times which is in accordance with the present scenario of accelerated expansion of the universe. It can also be seen that when $l = 1$ the shear scalar σ^2 vanishes so that our model becomes isotropic shear free.

4. Conclusions

From case-1 Cosmological models corresponding to viscous fluid distribution with trace free matter source in Brans-Dicke [6] theory of gravitation have been obtained, in this chapter a closed, open and flat FRW radiating viscous fluid models in five dimensions are represented by the models obtained. It is observed that all the physical quantities diverge at initial epoch and vanish for infinitely large values of cosmic time. The spatially homogeneous and isotropic universes in five dimensions just before compactification transition can be understood by the models. Also it is clear that from case-2, The Kantowski-Sachs space time in the scale covariant theory of gravitation postulated by Canuto et al. has been discussed. We have obtained the exact solutions of the field equations of this theory, when the sources of matter is a bulk viscous fluid with one dimensional cosmic strings, we got these by using (i) special law of variation for Hubble's parameter proposed by Berman [4], (ii) the shear scalar of the space time is proportional to scalar expansion and (iii) the fluid is barotropic so that $\bar{p} = p - 3\zeta H = \epsilon\rho$. It is observed that the model obtained is nonsingular, expanding and non-rotating. It is also observed that the model does not remain anisotropic throughout the universe evolution, so it shows a transition to an accelerated phase from a decelerated phase, at late time. This phenomenon is however is in agreement to the late time acceleration of the universe in modern cosmology. The various effects of the cosmic fluid may be a possible result of the removal of the initial anisotropies in our model. Added to this, in our model, there is a decrease in the bulk viscosity with an increase in cosmic time finally leading to an inflationary model.

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Appendix-I

Consider FRW metric in the form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\emptyset^2) + (1-kr^2)d\Psi^2\right]$$

Here Taking $x^0 = r,\ x^1 = \theta, x^2 = \emptyset, x^3 = \Psi, x^4 = t$

The metric tensor components are

$g_{00} = \frac{-a^2(t)}{1-kr^2},\quad g_{11} = -a^2(t)r^2,\quad g_{22} = -a^2(t)r^2 sin^2\theta,$

$g_{33} = -a^2(t)(1-kr^2),\quad g_{44} = 1$ and also $g_{ij} = 0$ for $i \neq j$

Conjugate metric tensor components are

$g^{ij} = \frac{G(i,j)}{g}$, *where* $g = |g_{ij}|$ & $G(i,j)$ is the expression formed by the cofactor of g_{ij} in the determinant $|g_{ij}|$.

Here $g = a^8 r^4 sin^2\theta$

Hence $g^{00} = \frac{-(1-kr^2)}{a^2(t)}, g^{11} = \frac{-1}{a^2(t)r^2}, g^{22} = \frac{-1}{a^2(t)r^2 sin^2\theta}, g^{33} = \frac{-1}{a^2(t)(1-kr^2)}, g^{44} = 1$

Non Zero Christoffel symbols of First kind:

Formula : $[i\,j,k] = \frac{1}{2}\left[\frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^k}\right]$

$[0\,0,0] = \frac{-ka^2r}{(1-kr^2)^2}$; $[0\,0,4] = \frac{a\dot{a}}{(1-kr^2)}$; $[1\,1,0] = a^2r$;

$[1\,1,4] = \dot{a}ar^2$; $[2\,2,0] = ra^2 sin^2\theta$; $[2\,2,1] = a^2r^2 sin\theta cos\theta$;

$[2\,2,4] = a\dot{a}r^2 sin^2\theta$; $[3\,3,0] = -ka^2r$; $[3\,3,4] = a\dot{a}(1-kr^2)$;

$[0\,4,0] = \frac{-a\dot{a}}{(1-kr^2)}$; $[1\,0,1] = -a^2r$; $[1\,4,1] = -a\dot{a}r^2$;

$[2\,0,2] = -ra^2 sin^2\theta$; $[2\,1,2] = -a^2r^2 sin\theta cos\theta$;

$[2\,4,2] = -a\dot{a}r^2 sin^2\theta$; $[3\,0,3] = ka^2r$; $[3\,4,3] = -a\dot{a}(1-kr^2)$

Non zero Christoffel symbols of Second kind:

Formula: $\left\{ {l \atop i\,j} \right\} = g^{lk}[i\,j,\ k]$

$\left\{ {0 \atop 0\,0} \right\} = g^{00}[0\,0,0] = \frac{kr}{(1-kr^2)}$

$\left\{ {4 \atop 0\,0} \right\} = g^{44}[0\,0,4] = \frac{a\dot{a}}{(1-kr^2)}$

$\left\{ {0 \atop 1\,1} \right\} = g^{00}[1\,1,0] = -r(1-kr^2)$

$\left\{\begin{matrix} 0 \\ 2\ 2 \end{matrix}\right\} = g^{00}[2\ 2, 0] = -r(1 - kr^2)sin^2\theta$

$\left\{\begin{matrix} 1 \\ 2\ 2 \end{matrix}\right\} = g^{11}[2\ 2, 1] = -sin\theta cos\theta$

$\left\{\begin{matrix} 4 \\ 2\ 2 \end{matrix}\right\} = g^{44}[2\ 2, 4] = a\dot{a}r^2 sin^2\theta$

$\left\{\begin{matrix} 0 \\ 3\ 3 \end{matrix}\right\} = g^{00}[3\ 3, 0] = kr(1 - kr^2)$

$\left\{\begin{matrix} 4 \\ 3\ 3 \end{matrix}\right\} = g^{44}[3\ 3, 4] = a\dot{a}(1 - kr^2)$

$\left\{\begin{matrix} 0 \\ 0\ 4 \end{matrix}\right\} = g^{00}[0\ 4, 0] = \frac{\dot{a}}{a}$

$\left\{\begin{matrix} 1 \\ 1\ 0 \end{matrix}\right\} = g^{11}[1\ 0, 1] = \frac{1}{r}$

$\left\{\begin{matrix} 1 \\ 1\ 4 \end{matrix}\right\} = g^{11}[1\ 4, 1] = \frac{\dot{a}}{a}$

$\left\{\begin{matrix} 2 \\ 2\ 0 \end{matrix}\right\} = g^{22}[2\ 0, 2] = \frac{1}{r}$

$\left\{\begin{matrix} 2 \\ 2\ 1 \end{matrix}\right\} = g^{22}[2\ 1, 2] = cot\theta$

$\left\{\begin{matrix} 2 \\ 2\ 4 \end{matrix}\right\} = g^{22}[2\ 4, 2] = \frac{\dot{a}}{a}$

$\left\{\begin{matrix} 3 \\ 3\ 0 \end{matrix}\right\} = g^{33}[3\ 0, 3] = \frac{-kr}{(1 - kr^2)}$

$\left\{\begin{matrix} 3 \\ 3\ 4 \end{matrix}\right\} = g^{33}[3\ 4, 3] = \frac{\dot{a}}{a}$

<u>Ricci tensor components:</u>

$R^{\alpha}{}_{,ijk} = \frac{\partial}{\partial x^j}\left\{\begin{matrix} \alpha \\ i\ \alpha \end{matrix}\right\} - \frac{\partial}{\partial x^\alpha}\left\{\begin{matrix} \alpha \\ i\ j \end{matrix}\right\} + \left\{\begin{matrix} \alpha \\ \beta\ j \end{matrix}\right\}\left\{\begin{matrix} \beta \\ i\ \alpha \end{matrix}\right\} - \left\{\begin{matrix} \alpha \\ \beta\alpha \end{matrix}\right\}\left\{\begin{matrix} \beta \\ i\ j \end{matrix}\right\}$

$R_{ij} = R^{\alpha}{}_{,ij\alpha} = -\frac{\partial}{\partial x^\alpha}\left\{\begin{matrix} \alpha \\ i\ j \end{matrix}\right\} + \frac{\partial}{\partial x^j}\frac{\partial}{\partial x^i}\left[log\sqrt{g}\right] - \left\{\begin{matrix} \beta \\ i\ j \end{matrix}\right\}\frac{\partial}{\partial x^\beta}\left[log\sqrt{g}\right] + \left\{\begin{matrix} \alpha \\ \beta\ j \end{matrix}\right\}\left\{\begin{matrix} \beta \\ i\ \alpha \end{matrix}\right\}$

(since $\left\{\begin{matrix} i \\ i\ j \end{matrix}\right\} = \left\{\begin{matrix} i \\ j\ i \end{matrix}\right\} = \frac{\partial}{\partial x^j}\left[log\sqrt{g}\right]$)

Now

$R_{00} = -\frac{\partial}{\partial x^\alpha}\left\{\begin{matrix} \alpha \\ 0\ 0 \end{matrix}\right\} + \frac{\partial}{\partial x^0}\frac{\partial}{\partial x^0}\left[log\sqrt{g}\right] - \left\{\begin{matrix} \beta \\ 0\ 0 \end{matrix}\right\}\frac{\partial}{\partial x^\beta}\left[log\sqrt{g}\right] + \left\{\begin{matrix} \alpha \\ \beta\ 0 \end{matrix}\right\}\left\{\begin{matrix} \beta \\ 0\ \alpha \end{matrix}\right\}$

$$\therefore R_{00} = -\left[\frac{\partial}{\partial x^0}\left\{\begin{matrix} 0 \\ 0\ 0 \end{matrix}\right\} + \frac{\partial}{\partial x^4}\left\{\begin{matrix} 4 \\ 0\ 0 \end{matrix}\right\}\right] + \frac{\partial}{\partial x^0}\frac{\partial}{\partial x^0}\left[log\sqrt{g}\right] - \left[\left\{\begin{matrix} 0 \\ 0\ 0 \end{matrix}\right\}\frac{\partial}{\partial x^0}\left[log\sqrt{g}\right] + \left\{\begin{matrix} 4 \\ 0\ 0 \end{matrix}\right\}\frac{\partial}{\partial x^4}\left[log\sqrt{g}\right]\right] + \left\{\begin{matrix} \alpha \\ 0\ 0 \end{matrix}\right\}\left\{\begin{matrix} 0 \\ 0\ \alpha \end{matrix}\right\} + \left\{\begin{matrix} \alpha \\ 1\ 0 \end{matrix}\right\}\left\{\begin{matrix} 1 \\ 0\ \alpha \end{matrix}\right\} + \left\{\begin{matrix} \alpha \\ 2\ 0 \end{matrix}\right\}\left\{\begin{matrix} 2 \\ 0\ \alpha \end{matrix}\right\} + \left\{\begin{matrix} \alpha \\ 3\ 0 \end{matrix}\right\}\left\{\begin{matrix} 3 \\ 0\ \alpha \end{matrix}\right\} + \left\{\begin{matrix} \alpha \\ 4\ 0 \end{matrix}\right\}\left\{\begin{matrix} 4 \\ 0\ \alpha \end{matrix}\right\}$$

$$R_{00}=-\left[\frac{\partial}{\partial r}\left\{\frac{kr}{(1-kr^2)}\right\}+\frac{\partial}{\partial t}\left\{\frac{a\dot{a}}{(1-kr^2)}\right\}\right]+\frac{\partial}{\partial r}\frac{\partial}{\partial r}\left[log\sqrt{g}\right]$$
$$-\left[\left\{\frac{kr}{(1-kr^2)}\right\}\frac{\partial}{\partial r}\left[log\sqrt{g}\right]+\left\{\frac{a\dot{a}}{(1-kr^2)}\right\}\frac{\partial}{\partial t}\left[log\sqrt{g}\right]\right]+\left\{{0 \atop 0\,0}\right\}\left\{{0 \atop 0\,0}\right\}$$
$$+\left\{{0 \atop 0\,4}\right\}\left\{{4 \atop 0\,0}\right\}+\left\{{1 \atop 1\,0}\right\}\left\{{1 \atop 0\,1}\right\}+\left\{{2 \atop 2\,0}\right\}\left\{{2 \atop 0\,2}\right\}+\left\{{3 \atop 3\,0}\right\}\left\{{3 \atop 0\,3}\right\}+\left\{{0 \atop 4\,0}\right\}\left\{{4 \atop 0\,0}\right\}$$

As $log\sqrt{g}=log(a^4.r^2.sin^2\theta)=4\log(a)+2\log(r)+2\log(sin\theta)$

$$R_{00}=-\left[\left\{\frac{k(1+kr^2)}{(1-kr^2)^2}\right\}+\left\{\frac{(a\ddot{a}+\dot{a}^2)}{(1-kr^2)}\right\}\right]-\frac{2}{r^2}-\left[\left\{\frac{kr}{(1-kr^2)}\right\}\frac{2}{r}+\left\{\frac{a\dot{a}}{(1-kr^2)}\right\}\frac{4\dot{a}}{a}\right]$$
$$+\left(\frac{kr}{(1-kr^2)}\right)^2+\frac{\dot{a}}{a}.\frac{a\dot{a}}{(1-kr^2)}+\frac{1}{r}.\frac{1}{r}+\frac{1}{r}.\frac{1}{r}+\left(\frac{-kr}{(1-kr^2)}\right)^2+\frac{\dot{a}}{a}.\frac{a\dot{a}}{(1-kr^2)}$$

$$R_{00}=\frac{-k}{(1-kr^2)^2}+\frac{k^2r^2}{(1-kr^2)^2}-\frac{3(\dot{a})^2}{(1-kr^2)}-\frac{a\ddot{a}}{(1-kr^2)}-\frac{2k}{(1-kr^2)}$$

Now $R_0^0=g^{00}R_{00}$

$$=\frac{-(1-kr^2)}{a^2(t)}.\left[\frac{-k}{(1-kr^2)^2}+\frac{k^2r^2}{(1-kr^2)^2}-\frac{3(\dot{a})^2}{(1-kr^2)}-\frac{a\ddot{a}}{(1-kr^2)}-\frac{2k}{(1-kr^2)}\right]$$

$$\therefore R_0^0=\frac{3(\dot{a})^2}{a^2}+\frac{\ddot{a}}{a}+\frac{3k}{a^2}$$

Similarly R_j^i For $i,j=1,2,3\&4$ $(i=j)$ are given by

$$R_1^1=\frac{3(\dot{a})^2}{a^2}+\frac{\ddot{a}}{a}+\frac{3k}{a^2}$$
$$R_2^2=\frac{3(\dot{a})^2}{a^2}+\frac{\ddot{a}}{a}+\frac{3k}{a^2}$$
$$R_3^3=\frac{3(\dot{a})^2}{a^2}+\frac{\ddot{a}}{a}+\frac{3k}{a^2}$$
$$R_4^4=\frac{4\ddot{a}}{a}$$

Now $R=R_0^0+R_1^1+R_2^2+R_3^3+R_4^4=\frac{12(\dot{a})^2}{a^2}+\frac{8\ddot{a}}{a}+\frac{12k}{a^2}$

<u>Einstein tensor components:</u>

Formula: $G_j^i=R_j^i-\frac{1}{2}R$

Now $G_0^0=R_0^0-\frac{1}{2}R=\left(\frac{3(\dot{a})^2}{a^2}+\frac{\ddot{a}}{a}+\frac{3k}{a^2}\right)-\frac{1}{2}\left(\frac{12(\dot{a})^2}{a^2}+\frac{8\ddot{a}}{a}+\frac{12k}{a^2}\right)$

$$=-\left(\frac{3(\dot{a})^2}{a^2}+\frac{3\ddot{a}}{a}+\frac{3k}{a^2}\right)$$

Similarly G_j^i For $i,j=1,2,3\&4$ $(i=j)$ are given by

$$G_1^1=-\left(\frac{3(\dot{a})^2}{a^2}+\frac{3\ddot{a}}{a}+\frac{3k}{a^2}\right)$$

$$G_2^2 = -\left(\frac{3(\dot{a})^2}{a^2} + \frac{3\ddot{a}}{a} + \frac{3k}{a^2}\right)$$

$$G_3^3 = -\left(\frac{3(\dot{a})^2}{a^2} + \frac{3\ddot{a}}{a} + \frac{3k}{a^2}\right)$$

$$G_4^4 = -\left(\frac{6(\dot{a})^2}{a^2} + \frac{6k}{a^2}\right)$$

Derivation of Field equations:

Formula:

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{\omega}{\emptyset^2}\left(\emptyset_{,i}\emptyset_{,j} - \frac{1}{2}g_{ij}\emptyset_{,k}\emptyset^{,k}\right) + \frac{1}{\emptyset}\left[\emptyset_{i;j} - g_{ij}\square\emptyset\right] = 8\pi\emptyset^{-1}T_{ij}$$

Multiplying above with contravariant tensor g^{il} then we get

$$R_j^i - \frac{1}{2}R = 8\pi\emptyset^{-1}T_j^l - \frac{\omega}{\emptyset^2}\left(\emptyset^{,l}\emptyset_{,j} - \frac{1}{2}\delta_j^l\emptyset_{,k}\emptyset^{,k}\right) - \frac{1}{\emptyset}\left(\emptyset^{,l}_{;j} - \delta_j^l\emptyset^{,k}_{;k}\right)$$

Since $G_j^i = R_j^i - \frac{1}{2}R$ and taking $l = i$

$$G_j^i = 8\pi\emptyset^{-1}T_j^i - \frac{\omega}{\emptyset^2}\left(\emptyset^{,i}\emptyset_{,j} - \frac{1}{2}\delta_j^i\emptyset_{,k}\emptyset^{,k}\right) - \frac{1}{\emptyset}\left(\emptyset^{,i}_{;j} - \delta_j^i\emptyset^{,k}_{;k}\right)$$

Where $\emptyset^{,k}_{;k} = \frac{\partial\emptyset^{,k}}{\partial x^k} + \emptyset^{,r}\left\{\begin{matrix} k \\ r\ \ k\end{matrix}\right\} = \frac{\partial\emptyset^{,k}}{\partial x^k}\emptyset^{,r}\left[\frac{\partial}{\partial x^r}\left(log\sqrt{g}\right)\right]$

$$\therefore G_j^i = 8\pi\emptyset^{-1}T_j^i - \frac{\omega}{\emptyset^2}\left(\emptyset^{,i}\emptyset_{,j} - \frac{1}{2}\delta_j^i\emptyset_{,k}\emptyset^{,k}\right) - \frac{1}{\emptyset}\left[\left(\frac{\partial\emptyset^{,i}}{\partial x^j} + \emptyset^{,r}\left\{\begin{matrix} i \\ r\ \ j\end{matrix}\right\}\right) - \delta_j^i\left(\frac{\partial\emptyset^{,k}}{\partial x^k} + \emptyset^{,r}\left[\frac{\partial}{\partial x^r}\left(log\sqrt{g}\right)\right]\right)\right]$$

For $i = 0$ & $j = 0$

$$G_0^0 = 8\pi\emptyset^{-1}T_0^0 - \frac{\omega}{\emptyset^2}\left(\emptyset^{,0}\emptyset_{,0} - \frac{1}{2}\delta_0^0\emptyset_{,k}\emptyset^{,k}\right) - \frac{1}{\emptyset}\left[\left(\frac{\partial\emptyset^{,0}}{\partial x^0} + \emptyset^{,r}\left\{\begin{matrix} 0 \\ r\ \ 0\end{matrix}\right\}\right) - \delta_0^0\left(\frac{\partial\emptyset^{,k}}{\partial x^k} + \emptyset^{,r}\left[\frac{\partial}{\partial x^r}\left(log\sqrt{g}\right)\right]\right)\right]$$

$\left(since \quad \delta_j^i = 1 \quad if\ i = j\right)$

$$= 8\pi\emptyset^{-1}T_0^0 - \frac{\omega}{\emptyset^2}\left(0 - \frac{1}{2}g^{kl}\emptyset_{,k}\emptyset_{,l}\right) - \frac{1}{\emptyset}\left[\left(0 + g^{rl}\emptyset_{,l}\left\{\begin{matrix} 0 \\ r\ \ 0\end{matrix}\right\}\right) - \left(\frac{\partial\emptyset^{,k}}{\partial x^k} + \emptyset^{,r}\left[\frac{\partial}{\partial x^r}\left(log\sqrt{g}\right)\right]\right)\right]$$

$$= 8\pi\emptyset^{-1}T_0^0 - \frac{\omega}{\emptyset^2}\left(0 - \frac{1}{2}g^{44}\emptyset_{,4}\emptyset_{,4}\right) - \frac{1}{\emptyset}\left[\left(0 + g^{44}\emptyset_{,4}\left\{\begin{matrix} 0 \\ 4\ \ 0\end{matrix}\right\}\right) - \left(\frac{\partial\emptyset^{,4}}{\partial x^4} + \emptyset^{,4}\left[\frac{\partial}{\partial x^4}\left(log\sqrt{g}\right)\right]\right)\right]$$

$$G_0^0 = 8\pi\emptyset^{-1}T_0^0 + \frac{\omega}{\emptyset^2}\left(\frac{1}{2}\left(\dot{\emptyset}\right)^2\right) - \frac{1}{\emptyset}\left[\left(\dot{\emptyset}\frac{\dot{a}}{a}\right) - \left(\ddot{\emptyset} + \dot{\emptyset}\left[\frac{4\dot{a}}{a}\right]\right)\right]$$

But Here $T_{ij} = (\rho + \bar{p})u_i u_j - g_{ij}\bar{p}$

Mixed form of above energy momentum tensor is

$$T_j^i = (\rho + \bar{p})u^i u_j - \delta_j^i\bar{p}$$

Hence

$$T_0^0 = (\rho + \bar{p})u^0 u_0 - \delta_0^0\bar{p} = -\bar{p} \quad \left(since \quad \delta_j^i = 1\ \ (if\ i = j)\ \&\ u^i u_j = 1\ if\ i = 3\right)$$

$$G_0^0 = -8\pi\emptyset^{-1}\bar{p} + \frac{\omega}{\emptyset^2}\left(\frac{1}{2}(\dot{\emptyset})^2\right) - \frac{1}{\emptyset}\left[\left(\dot{\emptyset}\frac{\dot{a}}{a}\right) - \left(\ddot{\emptyset} + \dot{\emptyset}\left[\frac{4\dot{a}}{a}\right]\right)\right]$$

But $G_0^0 = -\left(\frac{3(\dot{a})^2}{a^2} + \frac{3\ddot{a}}{a} + \frac{3k}{a^2}\right)$ then finally we get

$$\frac{3\ddot{a}}{a} + \frac{3\dot{a}^2}{a^2} + \frac{3k}{a^2} + \frac{\omega}{2}\left(\frac{\dot{\emptyset}}{\emptyset}\right)^2 - \frac{3\dot{a}\dot{\emptyset}}{a\emptyset} + \frac{\ddot{\emptyset}}{\emptyset} = 8\pi\emptyset^{-1}\bar{p}$$

Similarly we can find the remaining Field equations as follows

$$\frac{6\dot{a}^2}{a^2} + \frac{6k}{a^2} + \frac{\omega}{2}\left(\frac{\dot{\emptyset}}{\emptyset}\right)^2 + \frac{4\dot{a}\dot{\emptyset}}{a\emptyset} = 8\pi\emptyset^{-1}$$

$$\ddot{\emptyset} + \frac{4\dot{a}\dot{\emptyset}}{a} = 8\pi(3+2\omega)^{-1}(\rho - 4\bar{p})$$

$$\dot{\rho} + 4\frac{\dot{a}}{a}(\rho + \bar{p}) = 0$$

Appendix-II

The Kantowski-Sachs Metric is

$$ds^2 = dt^2 - E^2dx^2 - F^2(d\theta^2 + sin^2\theta d\emptyset^2)$$

Here Taking $x^1 = r,\ x^2 = \theta, x^3 = \emptyset,\ x^4 = t$

The metric tensor components are

$g_{11} = -E^2$, $g_{22} = -F^2$, $g_{33} = -F^2 sin^2\theta, g_{44} = 1$

and also $g_{ij} = 0$ for $i \neq j$

Conjugate metric tensor components are

$g^{ij} = \frac{G(i,j)}{g}$, *where* $g = |g_{ij}|$ & $G(i,j)$ is the expression formed by the cofactor of g_{ij} in the determinant$|g_{ij}|$.

Here $g = -E^2F^4sin^2\theta \Rightarrow \sqrt{-g} = EF^2 sin\theta$

$\therefore log\sqrt{-g} = logE + 2logF + \log(sin\theta)$

Hence $g^{11} = -\frac{1}{E^2}, g^{22} = -\frac{1}{F^2}, g^{33} = -\frac{1}{F^2sin^2\theta}, g^{44} = 1$

Nonzero Christoffel symbols of First kind:

Formula : $[i\,j,k] = \frac{1}{2}\left[\frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^k}\right]$

$[11,4] = E\dot{E}$; $[22,4] = F\dot{F}$; $[33,2] = F^2 sin\theta\, cos\theta$

$[33,4] = F\dot{F}sin^2\theta$; $[14,1] = -E\dot{E}$; $[24,2] = -F\dot{F}$

$[32,3] = -F^2 sin\theta\, cos\theta$; $[34,3] = -F\dot{F}sin^2\theta$;

Nonzero Christoffel symbols of Second kind:

Formula: $\left\{ {l \atop i\,j} \right\} = g^{lk}[i\,j,\ k]$

$\left\{ {4 \atop 11} \right\} = g^{44}[11,4] = E\dot{E}$

$\begin{Bmatrix} 4 \\ 22 \end{Bmatrix} = g^{44}[22, 4] = F\dot{F}$

$\begin{Bmatrix} 2 \\ 33 \end{Bmatrix} = g^{22}[33,2] = -sin\theta cos\theta$

$\begin{Bmatrix} 4 \\ 33 \end{Bmatrix} = g^{44}[33,4] = F\dot{F}sin^2\theta$

$\begin{Bmatrix} 1 \\ 14 \end{Bmatrix} = g^{11}[14, 1] = \frac{\dot{E}}{E}$

$\begin{Bmatrix} 2 \\ 24 \end{Bmatrix} = g^{22}[24,2] = \frac{\dot{F}}{F}$

$\begin{Bmatrix} 3 \\ 32 \end{Bmatrix} = g^{33}[32,3] = cot\theta$

$\begin{Bmatrix} 3 \\ 34 \end{Bmatrix} = g^{33}[34,3] = \frac{\dot{F}}{F}$

Ricci tensor components:

$R^{\alpha}{}_{,ijk} = \frac{\partial}{\partial x^j}\begin{Bmatrix} \alpha \\ i\ \alpha \end{Bmatrix} - \frac{\partial}{\partial x^\alpha}\begin{Bmatrix} \alpha \\ i\ j \end{Bmatrix} + \begin{Bmatrix} \alpha \\ \beta\ j \end{Bmatrix}\begin{Bmatrix} \beta \\ i\ \alpha \end{Bmatrix} - \begin{Bmatrix} \alpha \\ \beta\alpha \end{Bmatrix}\begin{Bmatrix} \beta \\ i\ j \end{Bmatrix}$

$R_{ij} = R^{\alpha}{}_{,ij\alpha} = -\frac{\partial}{\partial x^\alpha}\begin{Bmatrix} \alpha \\ i\ j \end{Bmatrix} + \frac{\partial}{\partial x^j}\frac{\partial}{\partial x^i}\left[log\sqrt{-g}\right] - \begin{Bmatrix} \beta \\ i\ j \end{Bmatrix}\frac{\partial}{\partial x^\beta}\left[log\sqrt{-g}\right] + \begin{Bmatrix} \alpha \\ \beta\ j \end{Bmatrix}\begin{Bmatrix} \beta \\ i\ \alpha \end{Bmatrix}$

(since $\begin{Bmatrix} i \\ i\ j \end{Bmatrix} = \begin{Bmatrix} i \\ j\ i \end{Bmatrix} = \frac{\partial}{\partial x^j}\left[log\sqrt{-g}\right]$)

Now

$R_{11} = -\frac{\partial}{\partial x^4}\begin{Bmatrix} 4 \\ 11 \end{Bmatrix} + \frac{\partial^2}{\partial x^{1^2}}\left[log\sqrt{-g}\right] - \begin{Bmatrix} 4 \\ 11 \end{Bmatrix}\frac{\partial}{\partial x^4}\left[log\sqrt{-g}\right] + \begin{Bmatrix} \alpha \\ \beta\ 1 \end{Bmatrix}\begin{Bmatrix} \beta \\ 1\ \alpha \end{Bmatrix}$

$$= -\frac{\partial}{\partial t}\left(E\dot{E}\right) + \frac{\partial^2}{\partial r^2}\left[log\sqrt{-g}\right] - \left(E\dot{E}\right)\frac{\partial}{\partial t}\left[log\sqrt{-g}\right] + \begin{Bmatrix} \alpha \\ 1\ 1 \end{Bmatrix}\begin{Bmatrix} 1 \\ 1\ \alpha \end{Bmatrix} + \begin{Bmatrix} \alpha \\ 2\ 1 \end{Bmatrix}\begin{Bmatrix} 2 \\ 1\ \alpha \end{Bmatrix} + \begin{Bmatrix} \alpha \\ 3\ 1 \end{Bmatrix}\begin{Bmatrix} 3 \\ 1\ \alpha \end{Bmatrix} + \begin{Bmatrix} \alpha \\ 4\ 1 \end{Bmatrix}\begin{Bmatrix} 4 \\ 1\ \alpha \end{Bmatrix}$$

$$= -\left[\left(\dot{E}\right)^2 + E\ddot{E}\right] + 0 - \left(E\dot{E}\right)\left[\frac{\dot{E}}{E} + 2\frac{\dot{F}}{F}\right] + \begin{Bmatrix} 1 \\ 14 \end{Bmatrix}\begin{Bmatrix} 4 \\ 11 \end{Bmatrix} + 0 + \begin{Bmatrix} 4 \\ 11 \end{Bmatrix}\begin{Bmatrix} 1 \\ 41 \end{Bmatrix}$$

$\therefore R_{11} = -E\ddot{E} - 2\frac{E\dot{E}\dot{F}}{F}$

Similarly $R_{22} = -\left(\dot{F}\right)^2 - F\ddot{F} - 1 - \frac{F\dot{F}\dot{E}}{E}$

$R_{33} = -sin^2\theta + \dot{F}^2 sin^2\theta - F\ddot{F}sin^2\theta - \frac{F\dot{F}\dot{E}}{E}sin^2\theta$

$R_{44} = \frac{\ddot{E}}{E} + 2\frac{\ddot{F}}{F}$

Similarly R^i_j For $i, j = 1,2,3\&4$ $(i = j)$ are given by

$R^1_1 = -\frac{\ddot{E}}{E} + 2\frac{\dot{E}}{E}\frac{\dot{F}}{F}$

$R^2_2 = \frac{\dot{F}^2}{F^2} + \frac{\ddot{F}}{F} + \frac{1}{F^2} + \frac{\dot{F}\dot{E}}{FE}$

$$R_3^3 = \frac{1}{F^2} - \frac{\dot{F}^2}{F^2} + \frac{\ddot{F}}{F} + \frac{\dot{F}\dot{E}}{FE}$$

$$R_4^4 = \frac{\ddot{E}}{E} + 2\frac{\ddot{F}}{F}$$

Now $R = R_1^1 + R_2^2 + R_3^3 + R_4^4 = 4\frac{\dot{E}}{E}\frac{\dot{F}}{F} + 4\frac{\ddot{F}}{F} + \frac{2}{F^2}$

Here the field equations are:

$$R_{ij} - \frac{1}{2}g_{ij}R + f_{ij}(\emptyset) = -8\pi G(\emptyset)T_{ij} + \Lambda(\emptyset)g_{ij}$$

Here $\emptyset^2 f_{ij} = 2\emptyset\emptyset_{i;j} - 4\emptyset_{,i}\emptyset_{,j} - g_{ij}(\emptyset\emptyset_{;jk}^{;k} - \emptyset^{,k}\emptyset_{,k})$

$R_j^i - \frac{1}{2}R + f_j^i(\emptyset) = -8\pi G(\emptyset)T_j^i$ (by neglecting $\Lambda(\emptyset)$)

But $f_j^i(\emptyset) = \frac{2}{\emptyset}\left[\frac{\partial\emptyset_{,i}}{\partial x^j} - \emptyset_{,r}\begin{Bmatrix} r \\ i \quad j\end{Bmatrix}\right] - \frac{4}{\emptyset^2}\emptyset_{,i}\emptyset_{,i} - \delta_j^i\left[\frac{1}{\emptyset}\left(\frac{\partial\emptyset^{,k}}{\partial x^k} + \emptyset^{,r}\begin{Bmatrix} k \\ r \quad k\end{Bmatrix}\right) - \frac{\emptyset^{,k}\emptyset_{,k}}{\emptyset^2}\right]$

Now $i = 1, j = 1$

$$f_1^1(\emptyset) = \frac{2}{\emptyset}\left[0 - \emptyset_{,4}\begin{Bmatrix} 4 \\ 1 \quad 1\end{Bmatrix}\right] - \frac{4}{\emptyset^2}\emptyset_{,1}\emptyset_{,1} - \left[\frac{1}{\emptyset}\left(\frac{\partial\emptyset^{,k}}{\partial x^k} + \emptyset^{,r}\begin{Bmatrix} k \\ r \quad k\end{Bmatrix}\right) - \frac{\emptyset^{,k}\emptyset_{,k}}{\emptyset^2}\right]$$

$$= \frac{2}{\emptyset}\left[0 - \emptyset_{,4}\begin{Bmatrix} 4 \\ 1 \quad 1\end{Bmatrix}\right] - \frac{4}{\emptyset^2}\emptyset_{,1}\emptyset_{,1} - \left[\frac{1}{\emptyset}\left(\frac{g^{lk}\partial\emptyset_{,l}}{\partial x^k} + \emptyset^{,r}\begin{Bmatrix} k \\ r \quad k\end{Bmatrix}\right) - \frac{g^{lk}\emptyset_{,l}\emptyset_{,k}}{\emptyset^2}\right]$$

$$= \frac{2}{\emptyset}\left[0 - \emptyset_{,4}\begin{Bmatrix} 4 \\ 1 \quad 1\end{Bmatrix}\right] - \left[\frac{1}{\emptyset}\left(\frac{g^{44}\partial\emptyset_{,4}}{\partial x^4} + \emptyset^{,r}\frac{\partial}{\partial x^r}\left[log\sqrt{g}\right]\right) - \frac{g^{lk}\emptyset_{,l}\emptyset_{,k}}{\emptyset^2}\right]$$

$$= -\frac{2}{\emptyset}\left(\dot{\emptyset}E\dot{E}\right) - \left[\frac{1}{\emptyset}\left(\ddot{\emptyset} + \dot{\emptyset}\left(\frac{\dot{E}}{E} + 2\frac{\dot{F}}{F}\right)\right)\right] - \left(\frac{\dot{\emptyset}}{\emptyset}\right)^2$$

Similarly

$$f_2^2 = -\frac{2}{\emptyset}\left(\dot{\emptyset}F\dot{F}\right) - \left[\frac{1}{\emptyset}\left(\ddot{\emptyset} + \dot{\emptyset}\left(\frac{\dot{E}}{E} + 2\frac{\dot{F}}{F}\right)\right)\right] - \left(\frac{\dot{\emptyset}}{\emptyset}\right)^2$$

$$f_3^3 = -\frac{2}{\emptyset}\left(\dot{\emptyset}F\dot{F}sin^2\theta\right) - \left[\frac{1}{\emptyset}\left(\ddot{\emptyset} + \dot{\emptyset}\left(\frac{\dot{E}}{E} + 2\frac{\dot{F}}{F}\right)\right)\right] - \left(\frac{\dot{\emptyset}}{\emptyset}\right)^2$$

$$f_4^4 = -\left[\frac{1}{\emptyset}\left(\ddot{\emptyset} + \dot{\emptyset}\left(\frac{\dot{E}}{E} + 2\frac{\dot{F}}{F}\right)\right)\right] - \left(\frac{\dot{\emptyset}}{\emptyset}\right)^2$$

And also $T_1^1 = \lambda - \bar{p}$, $T_2^2 = T_3^3 = -\bar{p}$, $T_4^4 = \rho$

Now by substituting the above Ricci tensor components, energy momentum tensor components and $f_j^{i'}s \quad for\ i = 1,2,3\&4$ then we get

$$2\frac{\ddot{F}}{F} + \frac{\dot{F}^2}{F^2} + \frac{1}{F^2} + \frac{\ddot{\emptyset}}{\emptyset} - \frac{\dot{\emptyset}^2}{\emptyset^2} - \frac{\dot{\emptyset}}{\emptyset}\left(\frac{\dot{E}}{E} - 2\frac{\dot{F}}{F}\right) = 8\pi G(\emptyset)(\bar{p} - \lambda)$$

$$\frac{\ddot{E}}{E} + \frac{\ddot{F}}{F} + \frac{\dot{E}}{E}\frac{\dot{F}}{F} + \frac{\ddot{\emptyset}}{\emptyset} - \frac{\dot{\emptyset}^2}{\emptyset^2} + \frac{\dot{E}}{E}\frac{\dot{\emptyset}}{\emptyset} = -8\pi G(\emptyset)\bar{p}$$

$$2\frac{\dot{E}}{E}\frac{\dot{F}}{F} + \frac{\dot{F}^2}{F^2} + \frac{1}{F^2} - \frac{\ddot{\emptyset}}{\emptyset} + 3\frac{\dot{\emptyset}^2}{\emptyset^2} + \frac{\dot{\emptyset}}{\emptyset}\left(\frac{\dot{E}}{E} + 2\frac{\dot{F}}{F}\right) = 8\pi G(\emptyset)\rho$$