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DYNAMIC MODELLING AND CONTROL OF TWO LINK FLEXIBLE ARM ROBOTIC MANIPULATOR

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Abstract

Robots are used in various fields now-a-days. The major application of robots is in industries. Increased rate of production within stipulated period of time along with high quality of products are the prime requirements of the industry. This is achieved by increasing the speed of operation. This increase in operation speed of robots causes problems of vibrations of *links* which are the major cause of positional inaccuracies at the end-effector. Furthermore, less power consumption is another area of concern. This may be achieved by decreasing the inertia of robots. But this results in lightweight *links* which are prone to vibrations. This further decreases the accuracy of robots. The present work is based on minimizing the vibrations of these lightweight robots also known as *flexible* robots. The thesis focuses upon the dynamic modeling and control of a Two-Link Flexible robot having two revolute *joints*. For this, firstly a mathematical model of the *flexible* robot is prepared using Lagrangian dynamics. The mathematical model thus obtained involves coupling between the *rigid* and *flexible* motions exhibited by the *flexible* robot. The *rigid* motion is due to the motion of *joints* and is responsible for change in configuration of the robot while the *flexible* motion is due to the vibration of *links*. The *links* undergo two types of vibrations: flexural/ bending vibrations and torsional vibrations. The vibration analysis of the *flexible links* is done using both *assumed modes method* and *finite elements method*. A robotic system is an inertia-variant system because its configuration changes with time. As a result, the natural frequencies of the system change with time. The effect of this time-dependency of natural frequencies of *links* on Joint and Tip responses is taken care of during mathematical modeling. While using *assumed modes method*, the governing equations of the system are obtained to include this effect. Exact boundary conditions for the *flexible links* are obtained while using this method. On the other hand, while using *finite elements method* this effect of time-dependency of natural frequencies is taken care of by continuously updating the mass and stiffness matrices of the system. Furthermore, it is also easy to take care of boundary conditions during 'finite element analyses'. Two different types of shape functions for a three-node frame element are proposed during the finite element formulation. Mass matrix, stiffness matrix and load vector are also derived for the same. Besides that, stiffness matrix for a one-dimensional torsional finite element under the influence of bending is also provided. Validation of the mathematical model of the Two-Link Flexible manipulator is done with the results available in literature and also through experiments. The control of vibrations of *flexible links* is achieved by using passive damping technique using viscoelastic material and active damping technique using piezoceramics. While using passive damping technique, the phenomenon of viscoelasticity is modeled using Kelvin-Voigt elements. The active vibration control of *flexible links* is achieved with the help of piezoelectric sensors and actuators applied in segmented fashion on the *links*. Direct velocity feedback is used. A hypothesis is presented for active vibration control of torsional vibrations. Results are also presented. Proportional-derivative gains are used during active vibration control and it is shown that good results are obtained at low values of control gains. To significantly reduce the vibration of *flexible links*, both the vibration control techniques are used together to achieve hybrid damping. Since, a robot is to be used for the performance of specified tasks, trajectory planning is must. In the present work, trajectory planning is done using both 'point-to-point' and 'continuous path' trajectories. It is shown that through proper planning of trajectory, tip vibrations, initial jerk and *joint* torque requirements can be reduced significantly. While making a robot follow a certain trajectory with minimum deviation from the desired path, control techniques are required. Computed-torque control and robust control techniques are used for this. Another control technique: Coupled-error dynamics control technique is also used in the present work. This is an innovative control technique discovered during the research work. It is based upon *independent modal space control* proposed by Meirovitch. The performance of this control technique is compared with computed-torque and robust control schemes in terms of error in path followed, effect of uncertainties within the system like mass uncertainty and *link* flexibility and control torque output. It is found that the performance of newly developed control scheme is better than computed-torque control and close to robust control.

Keywords: Flexible manipulator, dynamic analysis, vibration control

1. Introduction

Most of the robots used for industrial applications like welding, have two *links*. These robots are not perfectly rigid and hence possess some vibrations due to which the accuracy at the tip gets affected. The requirement of high performance in robotics asks for high speed of operation and good accuracy. Space applications demand for construction of large space structures by using lightweight space robot manipulator. All these requirements make it necessary to consider the structural flexibility in robotic arms during design stage. When compared with conventional *rigid* robots, elastic *link* manipulators have special potential advantages of higher operational speed and greater payload-to-manipulator weight ratio but, the flexibility of the arms leads to deformation and vibrations at the tips of the *links* during the motion. As a consequence, the dynamics behaviour becomes extremely complicated. Appearance of oscillations also makes the control problems really difficult. The present work is an attempt towards accurate modelling of a Two-Link Flexible manipulator as well as implementing different control strategies for the *flexible* manipulator. Mathematical model is prepared using Lagrangian dynamics to predict the behaviour of the manipulator. The *links* are modelled as Euler-Bernoulli beams. Discretization is done using assumed modes method (AMM) with variable frequency equation and Lagrangian-finite elements method (FEM) including torsional vibration modes. The control problem is divided into two parts,

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

viz., joint position control and vibration control of *links*. The *joint* position control is achieved using the actuators at the *joints*. For vibration control of *links*, both passive and active damping methods are used. Thus, damping is of hybrid nature. Few new proposals have also been made.

The aim of the research work is to achieve the position control of the end effector of a Two-Link Flexible robot using active and passive vibration control methods. The following objectives are established to realize this goal.

1. To prepare a dynamic model of a Two-Link Flexible manipulator considering both flexural and torsional vibrations of the *links*.
2. To control the position of the tip of the Two-Link Flexible manipulator using passive and active vibration control techniques
3. To perform the trajectory control of the Two-Link Flexible manipulator

1.1 Organization of the chapter

The complete work is divided into six sections. The **first section** gives a brief **introduction** of the present work. The **second section** provides the **literature review** of research in the area of *flexible* robotics. It highlights the different design and control concepts used by various researchers in the field of *flexible* robotics. The **third section** presents **mathematical modelling of Two-Link Flexible manipulator** using the approaches of Lagrangian-AMM and Lagrangian-FEM. In the **fourth section**, the **vibration control of *flexible links*** is described using viscoelastic, active and hybrid damping methods. In the **fifth section**, **trajectory control of Two-Link Flexible manipulator** is discussed. The trajectory control is achieved using both continuous path (CP) planning and point-to-point (PTP) planning methods. The position control of tip of *flexible* manipulator is achieved using the approaches of computed-torque control (CTC), robust control and coupled-error dynamics (CED). A comparison of these different approaches is made. The **sixth section** presents the **conclusions and recommendations** based on the present work.

2. Literature review

A comprehensive literature review was conducted to accomplish the aim and objectives of the present work. The literature review includes the survey on work done by various authors in the area of *flexible* robotics.

2.1 Literature survey on *flexible* robotics

The research on *flexible* manipulators started way back in year 1975 and is still going on. Dwivedy and Eberhard [1] have provided a comprehensive literature review on modelling and control of *flexible* manipulators. Table 2.1 highlights the major breakthroughs in this area in a chronological fashion.

Table 2.1: Major breakthroughs in the field of *flexible* robotics since 1975

S. No.	Year	Breakthroughs
1	1975-1980	Starting years of research on *flexible* robots; Focus on single *flexible link*; Dynamic modelling of *flexible* robots; Frequency-domain and time-domain analyses; Use of Lagrangian approach; Use of feedback control schemes
2	1981-1986	Dynamic modelling using Finite element method; Feedback control of vibrations; Inverse and Forward dynamics; Consideration of *joint* flexibility, Development of other control methods- singular perturbation, composite control, etc., Consideration of effect of gravity, Use of Newton-Euler approach in dynamic modelling
3	1986-1993	Use of nonlinear beam theory, Work started on nonlinear controller, Implementation of damping in dynamic modeling, Development of control strategy using Input shaping, Dynamic modelling of multilink *flexible* manipulators, Consideration of active damping using Linear quadratic regulator, Consideration of dynamic boundary conditions, Research on stability conditions, Feedforward control for gravity compensation
4	1994-2000	Hybrid force and position control, Adaptive control, Bending-torsion vibrations of a single *flexible* link, Comparison of AMM-FEM, Introduction of *flexible link* with prismatic joint, Stability characteristics considering effect of damping and tip mass, Adaptive nonlinear control, Effect of payload variations, Effect of geometric nonlinearities, Concept of Equivalent rigid-link system for dynamic modeling
5	2001-2008	Nonlinear Lyapunov control, Robustness and stability issues in model based control, Use of smart materials for active vibration damping, Fractional order controllers, Optimization techniques for controllers for vibration reduction, Redundant manipulators, Integral resonant control
6	2009 onwards	Control strategy based on intelligent techniques, Sliding mode control, Nonlinear observer, Intelligent control techniques, Robust input shaping, Spring-damper based mathematical model, Study of transient response, Study of effect of actuator on vibration modes

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

From the literature survey, it was found that different design methods have been used by various researchers for preparing the mathematical model of the *flexible* manipulators. Table 2.2 highlights the various design methods.

Table 2.2: Design methods used by various researchers for modelling the *flexible* manipulators

S.No.	Design Method	Researchers
1	Euler-Bernoulli beam theory	Book et al, 1975 [2]; DU et.al.,1996 [34]; Macchelli et. al.,2011 [63]; etc.
2	Spring-damper system	Zimmert and Sawodny, 2010 [56]
3	Timoshenko beam theory	Naganathan et al, 1986 [10]; Kermani,2010 [57]; Loudini,2013 [74]
4	Lagrangian dynamics and Finite Element Method	Sunada et al, 1981 [4]; Usoro et al, 1986 [11]; Bakr and Shabana, 1986 [12]; Bayo, 1987 [13]; Chedmail et al, 1991 [17]; Gaultier et.al.,1992 [21]; Stylianou and Tabarrok, 1994 [27], [28]; Zebin et.al.,2010 [59]; etc.
5	Controllable local degrees of freedom/ Redundant manipulator	Gao et. al.,2008 [46]; Bian et.al.,2009 [52]; Bian et.al.,2011 [68], etc.
6	Wave-based approach	O'Connor,2007 [44]; O'Connor et.al.,2009 [54]
7	Equivalent Rigid Link System	Vidoni et. al.,2013 [72] Gasparetto et. al.,2013 [73]
8	Lagrangian dynamics and Assumed modes method	Oakley et al, 1989 [15]; Luca et.al.,1991 [19]; Li and Sankar, 1993 [23]; Mayo et al, 1995 [33]; Lu et.al.,1996 [35]; Theodore and Ghosal, 1997 [36]; Ata et.al.,2012 [69]; Loudini,2013 [73]; etc.
9	Hamilton's principle	Gaultier et.al.,1992 [21], etc.

10	Newton-Euler dynamics and FEM	Nagnathan and Soni, 1986 [10]; Mohan and Saha [75]; etc.
11	Decoupled Natural Orthogonal Compliment	Mohan and Saha [75]

In order to achieve the tip position control of *flexible* manipulators, various control approaches have been used by the researchers. These are- Feed-forward control [56], [63], [71]; Integral resonant control [45], [53]; Input shaping [43], [58]; Sliding mode control [51], [55]; Model predictive control [66]; PD control [42], [74] and Lyapunov control [22], [39]. From the literature review it is observed that the most of the papers deal with planar single *link flexible* robotic arms with small 3D motions. For such *links*, a linear model is sufficient to describe the dynamic characteristics. A lot of research is going on, for non-linear models of *flexible* arms. The simple case of non-linearity in *flexible* arms is that of a *two-link* case which have been described in a few papers. Furthermore, *links* having revolute *joints* have been studied a lot. Most of the authors have focussed on the design of controllers according to the accurate dynamic model of the *flexible* arm. Both AMM and FEM have been used to model the *flexible* arm without compromising the accuracy. Some recent researchers have tried to use intelligent control techniques like fuzzy logic [59], [74], neural networks [31], [50], [71] and genetic algorithm [59], [74] for designing robust controllers as these do not require complicated mathematical modelling. There are two types of vibration control schemes: feed-forward and feedback. Trajectory control using *forward* and *inverse* dynamics methods [6], [7] and effect of gravity [24] on the motion of *flexible links* have been studied in a limited manner. Few authors recommend the use of nonlinear beam theory [14], [25] to account for geometric nonlinearities and a nonlinear controller [9], [31] for effective vibration control of *flexible* manipulators. In order to improve the dynamic performance of *flexible* manipulators, few optimization techniques have also been proposed. But, the most attractive one is the use of kinematic redundancy feature for minimizing *joint* torques and vibration suppression. The literature also gives a comparison between AMM and FEM [30], [32] and stability analysis of *flexible* manipulators. The proper placement of actuators made of smart materials on *flexible* links plays a crucial role in damping the vibrations [40]. The control approaches found in the literature apply well for the *single-link* planar case with small *elastic* displacements. For *multi-link* case, Feed-forward based and other advanced techniques might be suitable.

2.2 Research gaps identified in the existing knowledge

Following gaps are found in the literature review presented in this chapter:

1. Use of internal damping is considered in very few papers. Most of the papers do not consider damping during dynamic modelling of the *flexible* manipulator system. This important parameter affecting the response needs to be included.

2. Combined bending-torsion vibrations of Two-Link Flexible manipulator become very important and needs to be analyzed. In fact, this coupling increases a lot in *multi-link* situation.
3. There is ample scope of research on optimal placement of actuators according to the mode shapes for active damping. Although one paper presents a study on this area for bending vibrations, but is silent about the effect of *rigid* body motion on mode shapes.
4. Consideration of centrifugal and Coriolis' forces/ torques and gravity terms need to be considered while forming the equation of motion.
5. The control approaches used by various researchers are suitable for *single-link flexible* planar manipulator only and that too for small *elastic* displacements. For *multi-link flexible* manipulators, either feed-forward based techniques might be suitable or some different technique should be developed/ used.
6. A robotic arm as a plant is a time-varying system. Use of Adaptive Neural controllers or intelligent controllers is very limited. There exists scope for improvement using these.
7. Stability analysis of intelligent controllers is also another area of research on which hardly any literature is available.

3. Mathematical modelling of Two-Link Flexible manipulator

The mathematical modelling is done using both Lagrangian dynamics. Discretization is done using both AMM and FEM. For the AMM approach, the frequency equation is time-variant and this equation is evaluated at every time step.

3.1 Lagrangian dynamics for *flexible* manipulator

An accurate dynamic model of a Two-link Flexible manipulator having two revolute *joints* undergoing both small bending and small torsional deformations was prepared. Fig. 3.1 shows a Two-Link Flexible manipulator undergoing both bending and torsional deformations along with *rigid* revolutions at the *joints*.

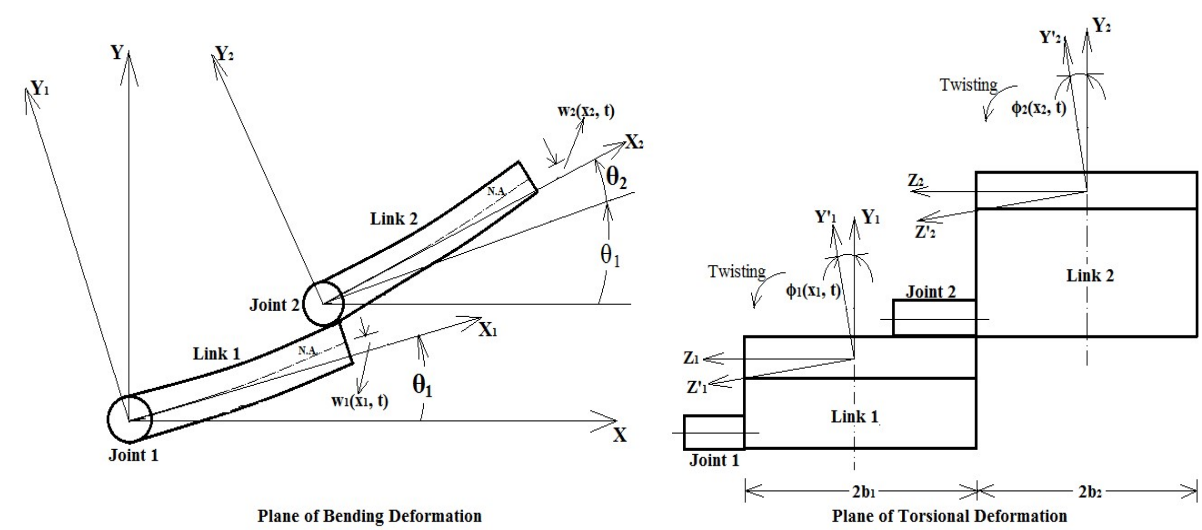


Fig. 3.1: Dynamic analysis of Two-Link Flexible manipulator undergoing both bending and torsional deformations.

In figure 3.1, plane X-Y is the plane of bending while plane Y-Z is the plane of torsion. X-Y-Z is the reference/ ground frame while X_1-Y_1-Z_1 and X_2-Y_2-Z_2 are the local frames attached to Link-1 and Link-2 respectively. Axis X_1 is aligned along the un-deformed neutral axis (N.A.) of Link-1 while axis X_2 is aligned along the un-deformed neutral axis of Link-2. The origins of these local frames are located at Joint-1 and Joint-2 respectively. Joint-1 is given a *rigid* rotation of θ_1 and Joint-2 is given a *rigid* rotation of θ_2. The position of any point on Link-1 with respect to ground is given by:

p1=T1T1Tr1T+T1r1 (3.1)

Similarly, the position of any point on Link-2 with respect to ground is given by:

p2=T1r1+T1T1Tr1T+TAT1T2r2+T1T2T2Tr2T (3.2)

In above expressions,

T1=cos1 −sin1 0 sin1 0 cos1 0 0 1 ; T2=cos2 −sin2 0 sin2 0 cos2 0 0 1 ; r1=x1 w1x1,t 0 ;

r1=L1 w1L1,t 0 ; r2=x2 w2x2,t 0 ; TA=cosw1' −sinw1' 0 sinw1' 0 cosw1' 0 0 1 ;

riT=0 bii bi ' ; riT=0 bii bi ' ; g'=0 g 0 ;

TiT=1 0 0 0 cosi −sini 0 sini cosi ; TiT=1 0 0 0 cosi −sini 0 sini cosi ; (3.3)

L_1 and L_2 = lengths of Link-1and Link-2 respectively,

θ_1 and θ_2 = *joint* rotations (rigid) of Joint-1and Joint-2 respectively,

x_1 and x_2 = distances measured along un-deformed Link-1 and Link-2 axes, i.e. X_1 and X_2 respectively,

$w_1(x_1, t)$ and $w_2(x_2, t)$ = *elastic* displacements of Link-1 and Link-2 respectively undergoing bending vibrations

w1'= bending angle at end point of Link-1 = dw1dx1

$\{r_1\}$ = position coordinates of any point on Link-1 w.r.t to un-deformed Link-1axis i.e., X_1 in plane X_1-Y_1

$\{r_2\}$ = position coordinates of any point on Link-2 w.r.t to un-deformed Link-2 axis i.e., X_2 in plane X_2-Y_2

$\{r_1^*\}$ = position coordinates of end point of Link-1 w.r.t. un-deformed beam-1 axis X_1 in plane X_1-Y_1

$\{r_{iT}\}$ = position coordinates of any point on Link-i in plane Y_i-Z_i

$\phi_i = \phi_i(x_i, t)$ = torsional deformation of any point on Link-i

$\phi_i^* = \phi_i(L_i, t)$ = torsional displacement of end point of Link-i; *i* represents the *link* number (*i* = 1 and 2)

The total kinetic energy of the manipulator system is given by:

K.E.=121A10L1p1´'p1´dx1+122A20L2p2´'p2´dx2 (3.4)

Total potential energy of the manipulator system is given by:

P.E.=E1I120L12w12x122dx1+G1J120L11x12dx1+1A10L1g'p1dx1+E2I220L22w22x222dx2+G2J220L22x22dx2+2A20L2g'p2dx2 (3.5)

In equation 3.5, J_1 and J_2 are the polar moment of inertias of Link-1 and Link-2 respectively. The *joint* torques can be obtained using Lagrangian dynamics as follows:

ddtLq´−Lq=F (3.6)

In above expression, L represents Lagrangian of the system and is obtained by taking the difference of total kinetic energy and total potential energy of the system; *q* represents generalized coordinates and *F* represents generalized torque/force.

q=1 2 w1 w1 w2 w2 1 1 2 2 ' (3.7a)

F=1 2 0 0 0 0 0 0 0 0 ' (3.7b)

where, 1 and 2 are the external torques applied at *joint-1* and *joint-2* respectively.

3.2 Assumed modes method

While using assumed modes method (AMM), only bending vibrations of *flexible links* are considered. The term 'w_i (x_i, t)' for any Link-*i* can be found out by the solution of equation of motion of Euler-Bernoulli beam. The equation is as follows.

2wit2+EiIiiAi4wixi4=fxi,t (3.8a)

The transient solution of above equation is given as follows.

wixi,t=n=1WnxiTnt (3.8b)

In equation 3.8, n = number of modes (n = 1, 2, ….∞); $W_n(x_i)$ = nth mode shape and is a function of distance 'x' measured along un-deformed beam axis for Link-*i*; $T_n(t)$ = time-dependent function of nth mode. Since it is impossible to include all the infinite number of modes of the system, hence it is modeled with reduced number of modes by assuming some definite number of modes which best describe the behavior of the system. Thus, we can rewrite the above equation with reduced number of modes say *m*, as follows:

wixi,t=n=1mWnxiTnt (3.9a)

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

In equation 3.9a, m = number of assumed modes. The boundary conditions used for the *flexible links* are time-dependent and given as follows:

i. pi2t2w20,txi EiIi2wi0,txi2=−J (3.10a)
ii. pi2wi0,tt2 EiIi3wi0,txi3=M (3.10b)
iii. EiIi2wiLi,txi2=−Jpi2t2wiLi,txi (3.10c)
iv. EiIi3wiLi,txi3=Mpi2wiLi,tt2 (3.10d)

In equations 3.10, the symbols have their usual meanings. The word ‘left’ means the left end of *link* and the word ‘right’ means the right end of the *link*. The frequency equation (equation 3.11) of any Link-*i* undergoing flexural vibrations is also derived and found to be varying with time.

z z coshzcosz zsinz+coszsinh−aibiz4−1=0 cosh coszsinhz−coshzsin−biz3 1+coshzcosz+aiz (3.11)

In equation 3.11, z=Li ; ai=MpiiAiLi ; bi=JpiiAiLi3. The expression for mode shape can be found out as follows:

Wnxi,t=C1ncoshnxi−cosnxi+nsinnxi−sinhnxi (3.12)

where, n=sinhnLi−sinnLi+ainLicoshnLi−cosnLicosnLi+coshnLi+ainLisinhnLi−sinnLi; n4=iAi EiIin2 ; ω_n = nth
mode natural angular frequency of the Link-*i*; C_{1n} = arbitrary normalization constant. The time-dependent term- ‘$T_n(t)$’ in equation (3.8b) is given by equation 3.13 as follows.

Tnt=e−nntAncosdnt+Bnsindnt+1iAibi0tQsinnt−dτ (3.13)

where, Q=0fxi,Wnxidxi
ξ_n = n^{th} mode damping ratio,
ω_n = n^{th} mode natural angular frequency,
ω_{dn} = n^{th} mode damped angular frequency = n1−n2.

The complete equation of motion of the Two-Link Flexible manipulator after considering the effect of payload is given as follows:

Mq6X6q´6X1+Hq,q´6X1+Gq6X1+Cq6X6q´6X1+Kq6X6q6X1+Kq6X1=Qt6X1 (3.14)

In equation 3.14, *M* stands for mass matrix, *H* stands for Coriolis and centrifugal torque vector, *G* stands for gravity torque vector, *C* stands for damping and gyroscopic couple matrix, *K* stands for stiffness matrix, $K^{\#}$ is the miscellaneous matrix that includes unmodelled dynamics and *Q* stands for torque vector. The vector *q* in equation 3.14 contains first six variables mentioned in equation 3.7a.

3.3 Finite element method

The finite element method (FEM) is used to model the vibratory motions of the *flexible links.* This involves division of *flexible links* into some finite number of elements and finding the inertia and stiffness matrices that govern the dynamics of the system under consideration. Figure 3.2 shows the discretization of *flexible links* using two Space-frame elements [76].

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

Fig. 3.2: Dynamics modelling of a Two-Link Flexible manipulator using two Space-frame finite elements.

A ‘space-frame element’ has two nodes with each node having six degrees of freedom: three translational (Q_{6i-5}, Q_{6i-4} and Q_{6i-3}) and three rotational (Q_{6i-2}, Q_{6i-1} and Q_{6i}) (refer to Fig. 3.2). The complete equation of motion of the *flexible* manipulator is given by equation 3.15a. The symbols have their usual meanings.

Mrr Mrf Mfr Mff n+NXn+Nqr´ qf´ n+NX1+Crr 0 0 Cff n+NXn+Nq´r q´f n+NXn+N+0 0 0 Kff n+NXn+Nqr qf n+NXn+N+H 0 n+NX1+G 0 n+NX1=Fr Ff n+NX1 (3.15a)

In equation 3.15a, subscripts- *r* and *f* stand for *rigid* and *flexible* respectively. *N* represents the *rigid* degrees of freedom present in the system and *n* represents the *flexible* degrees of freedom obtained from finite element formulation. For the present case, since there are two *flexible links*, we have *N* = 2. Hence, M_{rr} consists of two diagonal elements- M_{11} and M_{22}. M_{rf} and M_{fr} represent the coupling between *rigid* and *flexible* motions. It is also seen that

Mfr=Mrf' (3.15b)

3.4 Simulation results

Some typical simulation results based on models described earlier have been presented. Table 3.1 describes the physical and simulation parameters used for a Two-Link Flexible manipulator used by Habib and Korayem [77].

Table 3.1: Physical and simulation parameters for Two-Link Flexible manipulator [77]

Link parameters	**Value**
Length of links	$L_1 = L_2 = 0.5$ m
Width of links	$b_1 = 4$ cm; $b_2 = 5.17$ cm
Thickness of the links	$t_1 = 4$ mm; $t_2 = 1.5$ mm
Flexural rigidity of links	$EI_1 = 14.93$ Nm^2; $EI_2 = 1.017$ Nm^2
Mass per unit length of links	$\mu_1 = 0.504$ kg/m; $\mu_2 = 0.2442$ kg/m
Joint 1 torque (Fig. 3.3)	Square wave of amplitude 0.5 Nm and time-period 1 s
Joint 2 torque (Fig. 3.4)	Square wave of amplitude 0.1 Nm and time-period 1 s

The simulation results are shown in Fig. 3.5 to Fig. 3.7b. The natural frequencies and the general nature of response level matches with the predictions by Habib and Korayem [77].

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

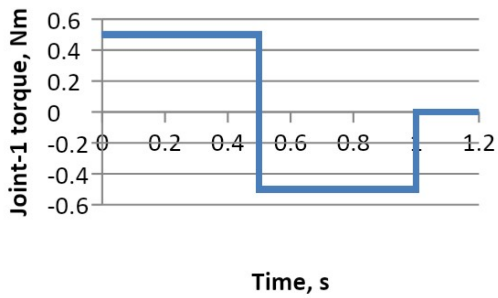


Fig.3.3: Torque applied at Joint-1

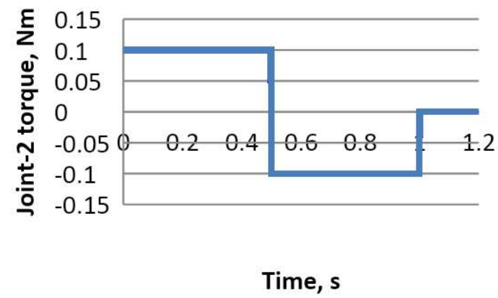


Fig.3.4: Torque applied at Joint-2

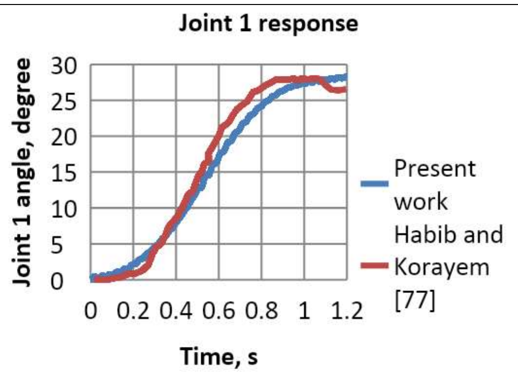


Fig.3.5: Comparison of Joint-1 response between present work and Habib and Korayem's work [77]

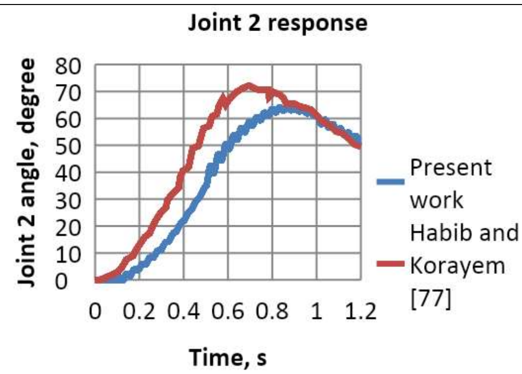


Fig. 3.6: Comparison of Joint-2 response between present work and Habib and Korayem's work [77]

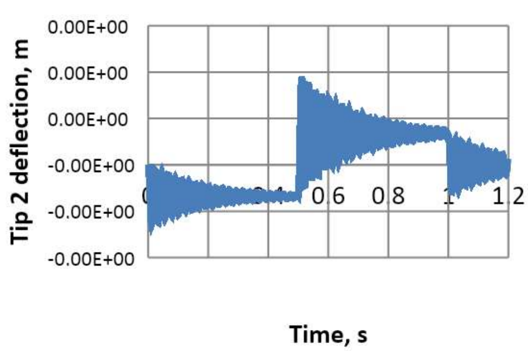


Fig.3.7a: Tip deflection of Link-2 as obtained in the present case using AMM (No. of modes used = 2; Modal damping ratio = 0.02 for each mode)

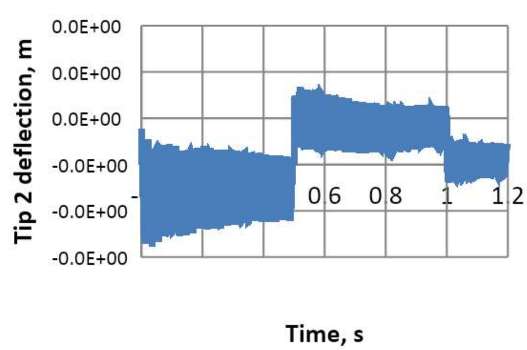


Fig.3.7b: Tip deflection of Link-2 as obtained in the present case using FEM (undamped case)

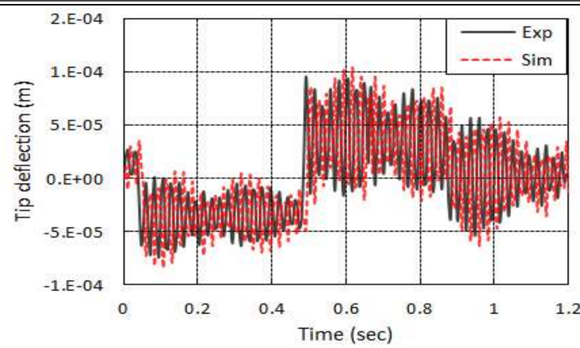


Fig.3.7c: Tip deflection of Link-2 as obtained by Habib [77]

4. Vibration control of *flexible links*

The vibration control of *flexible links* can be achieved in three ways: by passive viscoelastic damping, by active damping using piezoceramics and by hybrid damping. In viscoelastic damping method, a viscoelastic material (e.g., rubber) is pasted upon either throughout the *flexible link* or in the form of patches. Tzou and Wan [16] and Alberts et al [20] have used viscoelastic damping for controlling the vibrations of a *flexible* manipulator. In active damping, piezoelectric sensors and actuators are used for minimizing the vibrations of *flexible links*. The sensor-actuator pairs are applied in segmented fashion over the *flexible link*. In the third method of hybrid damping, both the viscoelastic damping and active damping are used together.

4.1 Vibration control using viscoelastic damping

The vibration control using viscoelastic damping involves the use of some viscoelastic material that is pasted over the *links*. Significant work has been done by Grootenhuis [78], Kapur et al [79], Dutt and Roy [80], etc. in the area of vibration control using viscoelastic materials. Adhikari and Woodhouse [81] tried to model the damping present within the structures. A prime contributor to viscoelastic damping is the shear strain within the viscoelastic material. The physical properties of these materials are found out to be frequency-dependent. These materials can be applied on the structure either in a constrained or unconstrained manner. Once applied, they become an integral part of the structure and provide a fixed damping behaviour. The mathematical modelling of viscoelastic damping is done using Kelvin-Voigt elements. For the Kelvin-Voigt element used for modelling the phenomenon of viscoelasticity, the stored energy and rate of dissipation in differential forms are given as follows [82]:

Storedenergy,dUe=0A12TdxdA (4.1a)

Rateofdissipation,de=0ADϵ2dxdA (4.1b)

In equations-4.1, σ = stress within the viscoelastic element of length *dx* and area *dA*, ϵ = strain within the viscoelastic element, η = dynamic viscosity of the viscoelastic material and *A* = total cross-sectional area of the viscoelastic patch pasted on the *flexible link*. The mass, stiffness and damping matrices for the viscoelastic material were found using FEM. The mass and stiffness matrices thus obtained were assembled with the mass and stiffness matrices of equation 3.15f. Damping matrix, $\mathbf{c}^{e'}$ for the viscoelastic material was derived and is as follows (equation 4.2):

ce'=c11e c12e c21e c22e (4.2a)

c11e=Aele 0 0 0 12Iele3 6Iele2 0 6Iele2 4Iele ; c12e=−Aele 0 0 0 −12Iele3 6Iele2 0 −6Iele2 2Iele ;

c21e=−Aele 0 0 0 −12Iele3 −6Iele2 0 6Iele2 2Iele ; c22e=Aele 0 0 0 12Iele3 −6Iele2 0 −6Iele2 4Iele

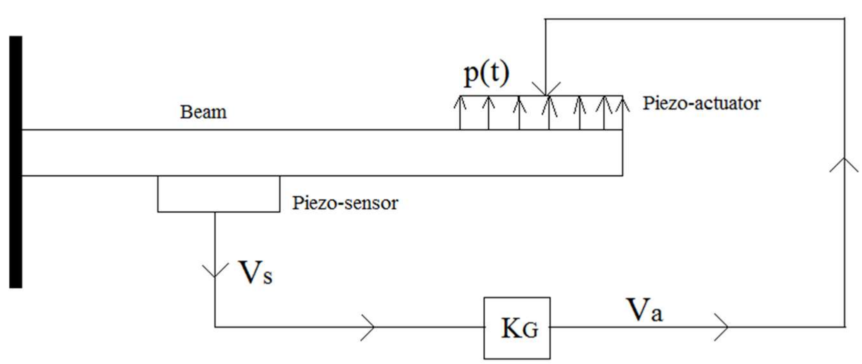
The damping matrix described in equation 4.2a is for a Plane-frame element. In case of torsion, the damping matrix was derived as follows:

ce'=Jele1 −1 −1 1 (4.2b)

Using matrices 4.2a and 4.2b, damping matrix C_{ff} for Space-frame element was formulated. In equations- 4.2a and 4.2b, A_e = area of cross-section of the element, l_e = length of the element, I_e = area moment of inertia of the element, J_e = polar area moment of inertia of the element and η = dynamic viscosity of the element.

4.2 Active vibration control using piezoceramics

A huge body of literature exists for vibration control of structures using piezoceramics, e.g., Cannon and Schmitz [83], Sakawa, Matsuno and Fukushima [84], Goh and Caughey [85], Chen et al [86], Sun et al [40], Preumont [88], etc. In active vibration control using piezoceramics, sensors and actuators are applied at suitable locations over the *flexible links*. When the sensor and actuator are at the same location, it is referred to as collocated arrangement and when the sensor and actuator are at different locations, it is known as non-collocated arrangement. Most of the researchers have used the approach of modal sensor and modal actuator [88] for active vibration control but in the present work, the vibration control is achieved by using direct feedback of velocity variables. Fig. 4.1 shows a smart beam having piezo-sensor and piezo-actuator applied on it. FEM is used to model the smart beam.



Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

Fig.4.1: Schematic diagram for active vibration control of a smart beam/ *link*. The piezo-actuator is modelled as a source of time-dependent uniformly distributed load, p(t). K_G is feedback gain.

The time-dependent uniformly distributed load, p(t) is added to the load vector F_f given by equation 3.15f. This is responsible for vibration damping of the *flexible links*. The voltage generated by piezo-sensor (considering it as a current amplifier) is given as follows:

vst=−RfEphd31abw´'"bpxdx (4.3)

where, R_f = piezo-resistance, w represents the deflection of any point on the beam and w' represents the slope at that point, b_p(x) = width of the piezo-sensor, *a* and *b* represent the initial and final coordinates of the points on the beam/*link* between which the piezo-sensor is located. These coordinates are measured along the beam axis, such that, '(b-a)' represents the length of the piezo-sensor. Taking b_p''(x) = constant, the voltage generated by the piezo-sensor will be expressed as follows.

vst=−RfEphd31.bpxw´'b−w´'a−bp'x.w´b−w´a (4.3b)

where, w´'b= rate of change of slope at point *b,* w´'a = rate of change of slope w.r.t. time at point *a* on the beam, w´b = rate of change of deflection at point *b* and w´a = rate of change of deflection at point *a* on the beam. The equation of motion for the beam with piezo-actuator is given by equation 4.4 as follows.

mw´+EIw''''=−Epd31vatbp''xh (4.4)

In equation 4.4, m = mass per unit length of the beam, EI = flexural rigidity of the beam, E_p = Young's modulus of the piezo-ceramic, h = thickness of the beam, w = deflection of the beam, v_a = voltage applied at the actuator and d_{31} = piezoelectric constant. Equation 4.4 is valid only when the thickness of the piezo-ceramic is negligible in comparison to the beam thickness. Fig. 4.2 shows the relative placement of sensors and actuators on the *flexible links* of the Two-Link Flexible manipulator.

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

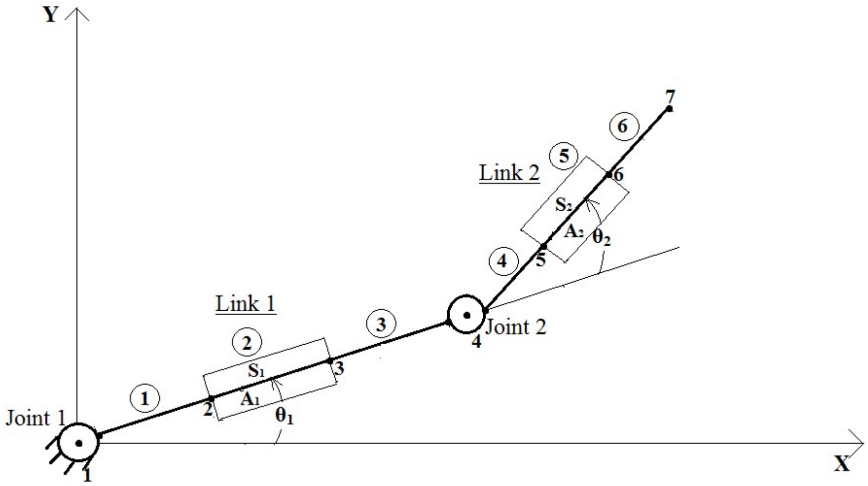


Fig. 4.2: Diagram showing the relative placements of sensors and actuators on the *flexible links* of the Two-Link Flexible manipulator. (In the figure, S_1 = Sensor on Link-1; S_2 = Sensor on Link-2; A_1 = Actuator on Link-1 and A_2 = Actuator on Link-2.)

As described before, we can have collocated and non-collocated arrangement for active vibration control of the *flexible links*. In Fig. 4.2, collocated arrangement of sensors and actuators are shown.

4.2.1 Use of piezoceramics for controlling torsional vibrations

In this section, mathematical models for piezo-sensor and piezo-actuator in torsion will be presented based upon the theory of piezoactuation provided by Preumont [88]. Using torsion equation we can write:

=TrJ and T=GJx=GJϕ' (4.5a)

Thus, =Gr'=Gγ (4.5b)

In above equations, = shear stress, *T* = twisting torque, *J* = polar moment of inertia, *G* = modulus of rigidity, = twist, = shear strain and *r* = radius (refer to Fig. 4.3). Taking the torsional piezo-sensor as current amplifier, the voltage, *v* generated by it can be expressed as:

v=Rd24Gprabbpx´'x.dx (4.6a)

where, *R* = resistance of piezo-sensor, d_{24} = piezoelectric constant, G_p = modulus of rigidity of piezo-sensor and b_p = width of piezo-sensor. If $b_p(x) = b_p$ = constant then,

v=Rd24Gprbp´'b−´'a (4.6b)

For torsional piezo-actuator we can write,

T=2E4423−d24vbpE44r.dA=GJ'−d24vE44bpr.dA (4.7)

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

where, E_{44} = Young's modulus of piezo-actuator, 23 = shear strain in piezo-actuator and A = area of cross-section of piezo-actuator.

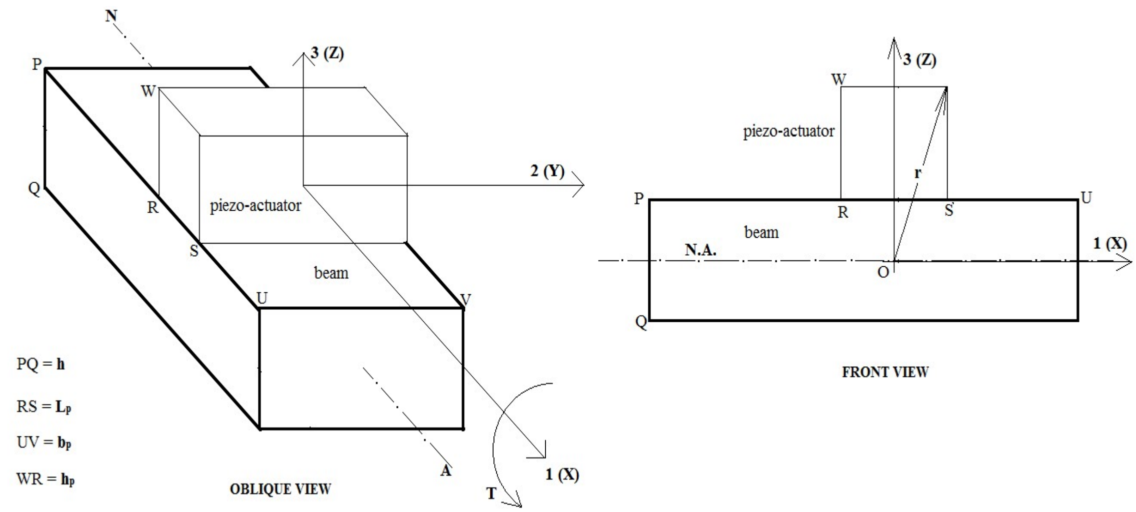


Fig.4.3: Development of mathematical model of torsional piezo-actuator

From Fig. 4.3 we can write,

r.dA=A0.5h+hp2+0.25bp2 (4.8)

where, $A = h_pL_p$, h = beam thickness, h_p = thickness of piezo-actuator and L_p = length of piezo-actuator. Using above equation, we can write:

T=GJ'−d24vE44hpLpbp.bp2.1+0.5h+hp0.5bp212 (4.9)

For $(0.5h+h_p) << b_p$, 0.5h+hp0.5bp21

So, T=GJ'−12d24vE44hpLp (4.10)

Now, inertia torque is represented as: Im´=−T

Therefore we get,

Im´+GJ'=12d24vE44hpLp (4.11)

where, I_m = mass moment of inertia of beam. It is known that shear stress is always complimentary. So, in order to control the torsional vibrations, two piezo-actuators should be attached on the opposite faces of the beam at the same location. Thus, above equation will get modified as follows.

Im´+GJ'=d24vE44hpLp (4.12)

4.3 Simulation results on vibration damping of *flexible links*

In this section, typical simulation results using viscoelastic damping, active damping and hybrid damping are discussed. Table 4.1 enlists the physical and simulation parameters used during simulation.

Table 4.1: Physical and simulation parameters for vibration damping of *flexible links* of Two-Link Flexible manipulator

Link parameter	**Value**
Length of *links*	$L_1 = L_2 = 0.5$ m
Width of *links*	$b_1 = 4$ cm; $b_2 = 5.17$ cm
Thickness of *links*	$t_1 = 4$ mm; $t_2 = 1.5$ mm
Flexural rigidity of *links*	$EI_1 = 14.93$ Nm^2; $EI_2 = 1.017$ Nm^2
Density of *links*	7850 kg/m^3
Joint 1 torque, tau1	A square wave of amplitude 0.5 Nm and frequency a. 100 Hz for Fig. 4.4, Fig. 4.6 and Fig. 4.7 b. 50 Hz for Fig. 4.5 and Fig. 4.8
Joint 2 torque, tau2	tau1 + A square wave of amplitude 0.1 Nm and frequency a. 100 Hz for Fig. 4.4, Fig. 4.6 and Fig. 4.7 b. 50 Hz for Fig. 4.5 and Fig. 4.8
Type of finite element used	Frame element/ Space-frame element
Dynamic viscosity of viscoelastic material (rubber)	1.8 Ns/m^2
Density of viscoelastic material (rubber)	1200 kg/m^3
Physical parameters of piezoceramic	$R_f = 1$; $E_p = E_{44} = 1$; $d_{31} = d_{44} = 1$

Simulation results are shown in Fig. 4.4 to Fig. 4.8. In figure 4.4, the undamped response and viscoelastically damped response of the tip of second *flexible link* are compared with each other.

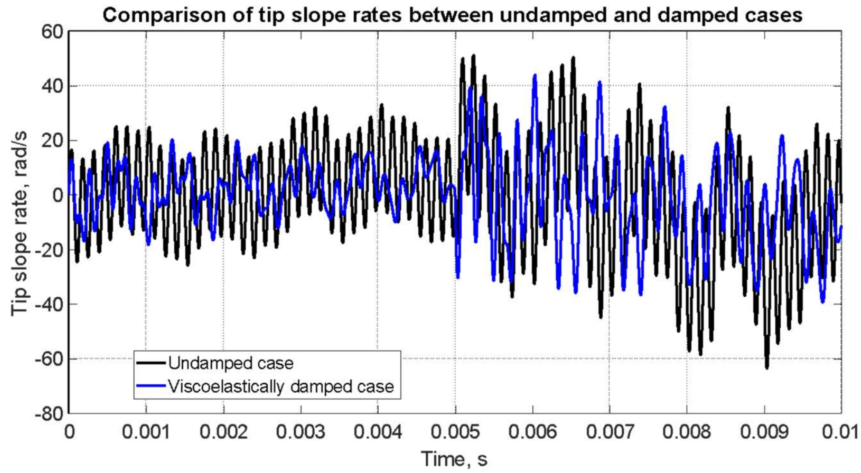


Fig. 4.4: Comparison of tip slope rates (rate of change of slope) of tip of second *flexible link* of Two-Link Flexible manipulator between viscoelastically damped and undamped cases

In Fig. 4.4, the rates of change of slope of tip of second *flexible link* for damped and undamped cases are compared. From the figure, it is clear that due to the presence of viscoelastic damping, the amplitude of vibration decreases. In Fig. 4.5, comparison of tip responses is shown for collocated and non-collocated arrangements. Descriptions about collocated and non-collocated control are already found within the literature [88].

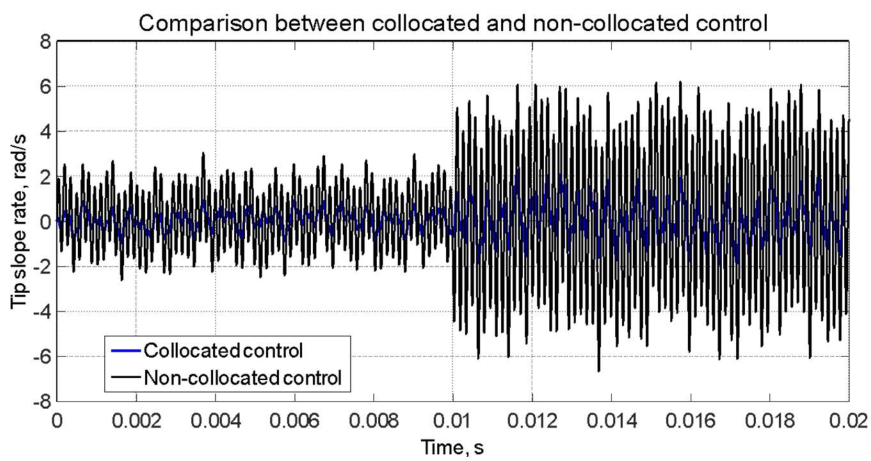


Fig. 4.5: Comparison of slopes of tip of second *flexible link* of Two-Link Flexible manipulator for collocated and non-collocated sensor-actuator pairs.

In Fig. 4.5, the slope rates of tip of second *flexible link* are compared at different arrangements of sensor-actuator pair. The values of proportional (K_p) and derivative (K_v) gains are taken as 4 for the both *links*. It is observed positional accuracy of collocated arrangement is the best. In figure 4.6, a comparison is done between the tip responses obtained by active damping and hybrid damping. During hybrid damping, both viscoelastic damping and active damping are used. In order to introduce viscoelastic damping, one thousand Kelvin-Voigt (K-V) elements are used.

China Petroleum Processing and Petrochemical Technology

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

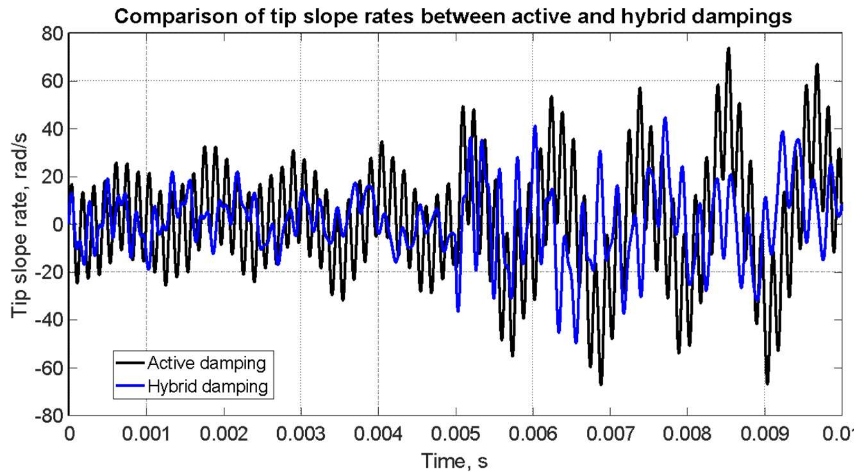


Fig. 4.6: Comparison of bending deflections of tip of second *flexible link* of Two-Link Flexible manipulator between active damping control and hybrid damping control.

From Fig. 4.6, it can be inferred that hybrid damping results in better vibration control. The positional accuracy described by zero vibration of the tips can be further increased by increasing the proportional gains- K_p. Fig. 4.7 shows the torsional deformations of tip of second *flexible link* under actively damped case and undamped case. Space-frame elements were used for obtaining this result.

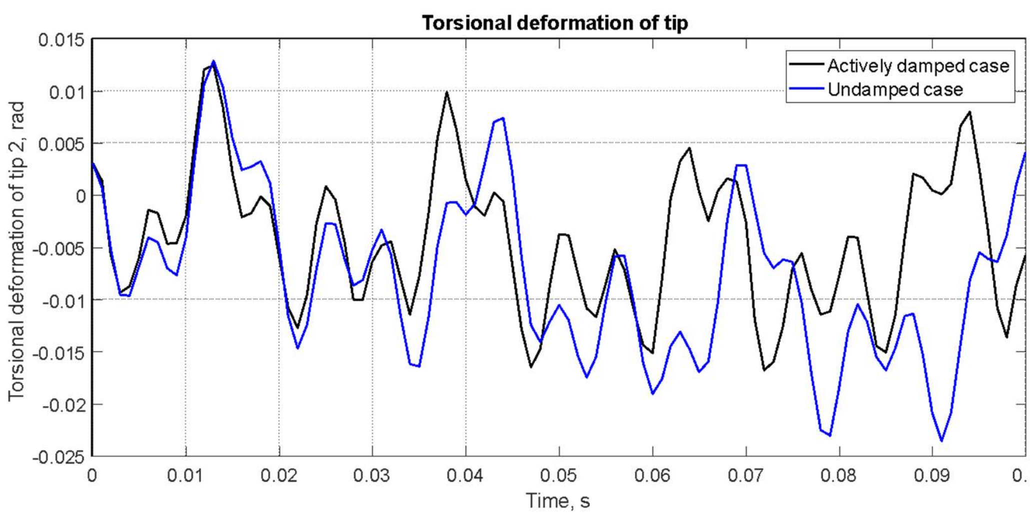


Fig. 4.7: Comparison of torsional deformations of tip of second *flexible link* of Two-Link Flexible manipulator between active damping control case and undamped case.

From Fig. 4.7, it can be observed that, the response of 'actively damped case' is closer to the zero line of reference than the response of 'undamped case'. The effect of vibrations on positional accuracy of tip of Two-Link Flexible manipulator is shown in Fig. 4.8.

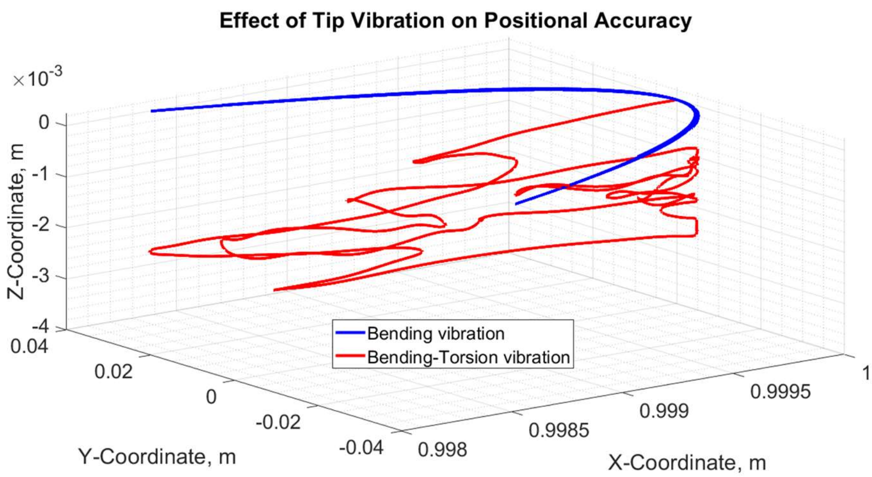


Fig. 4.8: Effect of vibration on positional accuracy of tip of Two-Link Flexible manipulator

In Fig. 4.8, the blue-coloured curve corresponds to the tip position of *flexible* manipulator when only bending vibration is considered. It can be seen that the curve lies in X-Y plane. The red-coloured curve corresponds to the tip position of *flexible* manipulator when both bending and torsional vibrations are considered. The curve does not remain in plain and the positional accuracy of the tip is also deteriorated.

5. Trajectory control of Two-Link Flexible manipulator

In this chapter, the position of tip of the *flexible* manipulator will be controlled by controlling the torque provided by the motors at the *joints*. For this purpose, it is required to perform the trajectory planning. A *flexible* manipulator exhibits two kinds of motion- 'rigid motion' and 'elastic motion'. Trajectory planning makes use of the 'rigid motion'. It can be done in two ways- continuous path (CP) planning and point-to-point (PTP) path planning. Trajectory can be defined either by using Joint-space trajectory or by using Cartesian-space trajectory. This trajectory planning forms the basis of manipulator control problem [90] which is stated as:

To find the joint actuator torques required to produce a planned trajectory, i.e., location, velocity and acceleration, for the entire work cycle such that planned task is performed as specified.

Even though the trajectory is planned properly, still there will be positional inaccuracy at the tip of the *flexible* manipulator. This is due to the 'elastic motion' exhibited by the *flexible links*. In the present case, the 'elastic motion' is modelled as a non-linearity and its effect is minimized by using three control techniques: computed torque control (CTC) [91], robust control [91] and a newly developed scheme called as coupled-error dynamics (CED).

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

5.1 Coupled-error dynamics

In this technique developed, the dynamics of the system is described in terms of *joint*-angle errors. Fig. 5.1 shows a Two-Link Rigid manipulator having a payload at its end. The dynamics equation of such a manipulator is given as follows:

1 2 =M11 M12 M21 M22 ´1 ´2 +N1 N2 +G1 G2 (5.1)

where, τ_1 and τ_2 are *joint* torques, θ_1 and θ_2 are *joint* angles and

M11=m1+m2L12+m2L22+2m2L1L2cos2+Ih1+MpL12+L22+2L1L2cos2+mh2L12

M12=M21=m2L22+m2L1L2cos2+MpL1L2cos2+L22

M22=m2L22+Ih2+MpL22

N1=−m2L1L22´1´2+´22sin2+MpL1L22´1´2−´22sin2

N2=m2L1L2´12sin2−2MpL1L2sin2.´1´2

G1=m1+mh2+m2+MpgL1cos1+m2+MpgL2cos1+2

G2=m2+MpgL2cos1+2

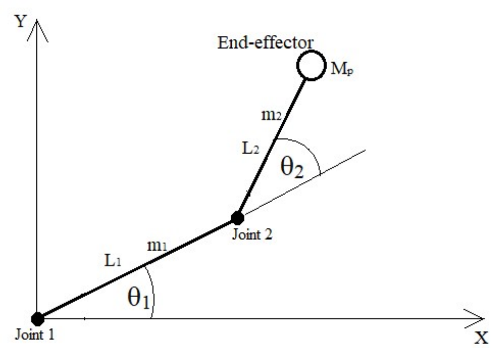


Fig.5.1: A Two-Link Rigid serial robot having two Revolute Joints in X-Y plane.

In equation 1, m_1 and m_2 are masses of Link-1 and Link-2 respectively, L_1 and L_2 are lengths of Link-1 and Link-2 respectively, I_{h1} and I_{h2} are *joint/hub* mass moment of inertias at Joint-1 and Joint-2 respectively, m_{h2} is the mass of hub of Joint-2 and M_p is mass of payload attached at the end of Link 2, i.e., at the end-effector. It can be seen from equation 5.1 that the mass matrix is coupled and depends upon configuration of the manipulator. This makes the control problem difficult. Now, a robot is designed to move along a desired trajectory for performing an assigned task. Thus, the desired *joint* angles are: θ_{1d} for Joint 1 and θ_{2d} for Joint 2. The *joint* errors will be defined as follows.

je=jd−j (5.2)

Catalyst Research Volume 23, Issue 2, October 2023 Pp. 2153-2181

where, *j* represents joint number (=1, 2), subscript- 'e' stands for 'error' and subscript- 'd' stands for 'desired'. After substituting for θ_1, θ_2 and their derivatives from equation 5.2 into equation 5.1 and performing necessary mathematical operations, following equations are obtained:
1d−1 2d−2 =Me11 Me12 Me21 Me22 ´1e ´2e −Ce11 Ce12 Ce21 Ce22 ´1e ´2e −G11e G12e G21e G22e 1e 2e (5.3a)

Defining the control torque as:

u==Kv´e+Kpe (5.4)

for each *joint* of the manipulator and substituting in eq. (5.3a), we get

Me´e+Kv−Ce´e+Kp−Gee=d (5.3b)

In eq. (5.3b), the matrices- $\mathbf{M_e}$, $\mathbf{C_e}$ and $\mathbf{G_e}$ are time-varying and also involve non-linear terms. It refers to the case of *coupled control* [92]. The values of gains- $\mathbf{K_v}$ and $\mathbf{K_p}$ must lie within certain range so that the controller performance may not deteriorate even though the coefficients in (5.3b) change with time. The ranges for $\mathbf{K_v}$ and $\mathbf{K_p}$ can be found out by designing the controller for minimum and maximum values of d and ´d. After applying the concept of *independent modal-space control* (IMSC) [92] on equation 5.3b, the relationship between $\boldsymbol{K_v}$ and $\boldsymbol{K_p}$ for critically damped response of the controller can be found out as follows:
Kv−Ce=1Me+2Kp−Ge (5.5)
where, 1 and 2 are some constants that depend upon the inertial and elastic properties of the manipulator system. In case of CTC, each joint is controlled separately and the relationship between controller gains is given as follows:
Kv=2Kp (5.6)

5.2 Simulation results for trajectory control
Performances of three types of controllers based on CTC, robust control and CED are compared through the simulation results. The control task is that the tip of the *flexible* manipulator should trace a straight line in X-Y plane described below:

$y=0.268x+0.5$ (5.7)

The *joints* of the manipulator follow a five-degree polynomial trajectory. The simulation results are presented below. Same values of controller gains were used during simulation for comparing the performances of the three controllers. Fig. 5.2 compares between the tip positions of the *flexible* manipulator as obtained by using controller based upon CTC scheme and CED scheme. Fig. 5.3 compares between the tip positions of the *flexible* manipulator as obtained by using controller based upon robust control scheme and CED scheme. The control torque requirement at Joint 1 is shown and compared with each other for the three controllers in Fig. 5.4.

Fig. 5.2: Comparison between the tip positions of *flexible* manipulator as obtained by using CTC and CED for same PD gains

Fig. 5.3: Comparison between the tip positions of *flexible* manipulator as obtained by using robust control and CED for same PD gains

Fig. 5.4: Comparison of control torque requirements at Joint 1 as provided by controller based upon CTC, robust control and CED for same PD gains (The control torque in 'CED' case lies between -0.7 units to 0.1 units.)

After looking at Fig. 5.2 and Fig. 5.3, it can be concluded that the performance of CED-based controller lies between that of the CTC and the robust controller for same values of controller (PD) gains. When Fig. 5.4 is viewed, it can be inferred that the CED-based controller gives the output comparable to the other controllers at minimum control torque.

6. Conclusions and recommendations

In this chapter, conclusions based upon the present work are made and new proposals are suggested.

6.1 Conclusions

The objectives of dynamics modelling, vibration control and trajectory control of a Two-Link Flexible robot have been accomplished in this work. While preparing the dynamic model, internal damping using the phenomenon of viscoelasticity has been considered. Combined bending and torsional vibrations of the *flexible links* were considered during mathematical modelling. Both AMM and FEM were used for mathematical modelling. The boundary conditions are easily addressed by using the FEM approach. In the present case, the differential equations obtained as a result of finite element (FE) formulation were solved directly to find out the responses at each node. The uniqueness lies in solving the equations of *rigid* and *elastic* motions for the *flexible links* simultaneously using a single computer program. During mathematical modelling, it was observed that the Two-Link Flexible manipulator exhibits time-varying boundary conditions. As a result, the frequency equation and hence the mode shapes also become time-dependent. This time-variance is due to the change in variables representing 'rigid motion' of the *flexible* manipulator with time. The non-linear effects due to the presence of centrifugal and Coriolis terms and gravity are also included in the formulation.

The process of active vibration control is based on direct feedback control using PD controller. The effect of relative position of sensors and actuators located on *flexible links* has been observed. It was found that collocated arrangement provides better positional accuracy than the non-collocated arrangement. The latter may provide better response when there is high internal damping within the system. The trajectory control of the *flexible* manipulator was achieved using three techniques, *viz.* CTC, robust control and CED. Out of these, the first two are well established techniques while the third one was developed during the research work. The mathematical model for active control of torsional vibrations has already been described.

6.2 Future recommendations

Literature provides three methods of mathematical modelling of a mechanical system- Lagrangian dynamics, Newton-Euler approach and Kane's method. The computational efficiencies of the computer programs based on these three methods can be checked. The conventional approach of active vibration control makes use of modal sensors and actuators. Literature describes the control problems associated with these, like residual modes and spillover effects. In the present work, direct feedback is used and the concept of modal sensor and actuator is not used. There is need of finding the control problems associated with this approach. Use of intelligent controllers and stability analysis of various controllers need to be done.

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