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ANALYSIS OF SOME INVENTORY MODELS TO SMOOTH INVENTORY MANAGERS OF ACCURATELY FORECASTING AND MANAGING TIME: DEPENDENT DEMANDS

Prashant Sharma[1], Birendra Kumar Chauhan[2], Gajraj Singh[*]

[1]Research Scholar, Department of Mathematics,
School of Basic Science and Technology, IIMT University,
Ganga Nagar, Meerut, U.P-250001 India
prashant1212sharma@gmail.com

[2]Professor, Department of Mathematics, School of Basic Science and Technology,
IIMT University, Ganga Nagar, Meerut, U.P-250001 India
birendrakumar_as@iimtindia.net

[*]Assistant Professor, Discipline of Statistics, School of Sciences,
Indira Gandhi National Open University, Delhi-110068 India
gajraj76@gmail.com
[*]Corresponding Author: - Gajraj Singh

Abstract: Inventory management plays a crucial role in ensuring the smooth operation of business across various industries. One of the key challenges faced by inventory managers is accurately forecasting and managing time i.e. dependent demand. The research paper presents a comprehensive analysis of inventory models that addresses to the complexities associated with time dependent demands patterns. The study is to provide an overview of the existing literature, explore different approaches to inventory modelling and identify the strengths and limitations of various techniques. Through the systematic review of scholarly modelling to the better cope with time–dependent demands and concludes by discussing future research directions and potential areas of improvement in the field of inventory management.
Keywords: *Inventory, Time dependent, Management, Production, Inventory Modelling.*

Introduction: Inventory is an important ingredient of any types of business and it is controlled by unreliable, inefficient and less cost effective. Most of the necessary goods like cosmetics items, radio-active substances, fashion goods, pharmaceuticals, food items etc. It is decrease due to its level or inventory continuously decreases and thereby, deterioration occurs. To get the actual inventory cost, this deterioration must be taken into consideration. On the other hand to make the inventory model cost at optimum level i.e. to get the minimum inventory cost a suitable inventory model is required which suits to meet the actual demand in the market. In minimizing inventory cost. This paper proposes a time dependent inventory model with constant production rate and exponential demand of materials with small amount of decay, whereas the existing models very often ignore the production rate, instead those consider the instantaneous replenishment rate. In periods of economics crisis, companies have to reduce costs at all stages of the production-storage distribution system. The decreases in costs related to the activities of tory control and management is a fundamental activity of organization and it is vital for the development of product sales and the marketing process.

Background and Significance: Effective inventory management is a critical aspect of supply chain operations, enabling businesses to meet customer demands while minimizing costs and maximizing profitability. In real-world scenarios, the demand for products often fluctuates over time, making it essential to develop inventory models that account for these time-dependent demands. Traditionally, inventory models assumed a constant demand rate, which does not accurately reflect the dynamic nature of real-world markets. Time-dependent demands can arise due to various factors such as seasonal variations, promotional campaigns, product lifecycles, market trends, and external events. Failing to consider these variations can lead to suboptimal inventory levels, resulting in stock outs, excess inventory, increased holding costs, and ultimately, dissatisfied customers. To address the challenges posed by time-dependent demands, researchers and practitioners have developed specialized inventory models that incorporate forecasting techniques, lead time considerations, and optimization methods. These models aim to strike a balance between maintaining sufficient inventory levels to meet customer demand and minimizing costs associated with inventory holding, ordering, and stock outs. The analysis of developed inventory models with time-dependent demands is crucial for several reasons. Firstly, it provides insights into the dynamics of inventory management in realistic scenarios, allowing businesses to make informed decisions about their inventory policies and strategies. Secondly, by incorporating accurate demand forecasting and optimization techniques, these models enable companies to enhance their supply chain performance and gain a competitive edge in the marketplace. Thirdly, a thorough analysis of existing models helps identify gaps and areas for improvement, paving the way for further research and the development of more effective inventory management solutions. By conducting a comprehensive analysis of developed inventory models with time-dependent demands, this research aims to contribute to the existing body of knowledge in the field of inventory management. The findings of this study can assist practitioners in implementing effective inventory policies, optimizing resource allocation, and improving customer satisfaction. Moreover, the research outcomes can guide future research endeavours, leading to the refinement and development of more sophisticated inventory models to address the challenges posed by time-dependent demands. In conclusion, the analysis of developed inventory models with time-dependent demands is a significant area of research that has practical implications for businesses. By understanding and utilizing these models, organizations can enhance their inventory control strategies, minimize costs, and improve overall supply chain performance in dynamic market environments.

Literature Review: Adequate records of effort have previously been completed by a huge amount of investigators in the region of manufacture catalogue prototypical to figure the appropriate inventory models. In last few years, many investigators have calculated in this field and developed inventory models to resolve the actual life problematic. In conventional record models, the request rate but on ground, it must not continuously correct. [1] Baker, R.C.and Urban, T. L., presented a model of deterministic inventory system with an inventory level dependent demand rate. [2] Chang, C. T. Inventory model with stock-dependent demand and non-linear holding costs for deteriorating items. [3] Chung, K. J An algorithm for an inventory model with inventory-level dependent demand rate. [4] Chang, C. T., Tang, J. T. and Goyal, S. K. Optimal replenishment policies for non - instantaneous deteriorating items with stock -dependent demand. [5] Chung, K. J., and Huang, J. S. The optimal retailers ordering policies for deteriorating items with limited storage capacity under trade credits [6] Dye, C. Y. and Ouyang, L.Y. an EOQ model for perishable items under stock dependent selling rate and time-dependent partial backlogging. [7] Datta T. K., and Pal, A. K. a note on an inventory model with inventory level dependent demand rate. [8] Goyal, S. K., and Chang C. T. optimal ordering and transfer policy for an inventory with stock dependent demand. [9] Huang, Y. F.An inventory model under two level of trade credit and limited storage space delivered without deliveries. [10] Jiangtao M. Guimei C. Teng, F. and Hong, M., presented a model Optimal ordering policies for permissible multi-item under stock-dependent demand and two-level trade credit. [11] Levis P. I. McLaughlin C. P. Lamone R. P. and Kottas, J. F Production Operation Management: Contemporary Policy for Managing Operating System.[12] Min, J., Zhou, Y. W., and Zhao, J., presented a model "An inventory model for deteriorating items under stock dependent demand and two-level trade credit. [13] Urban, T. L., and Baker, R. C., Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns. [14] Pal, M., and Chandra S. a Periodic review inventory model with stock-dependent demand permissible delay in payment and price discount on backorders. [15] Soni, H. N., and Shah, N. H. Optimal ordering policy for stock-dependent demand under Progressive payment scheme. [16] Soni, H. N. Optimal replenishment policies for non-instantaneous deteriorating it with price stock-sensitive demand under permissible delay in payments. [17] Ray, J., and Chaudhuri, K. S. an EOQ model with stock dependent-demand, shortage, inflation and time discounting. [18] Triatic, R. P., and Singh D. Inventory model with stock-dependent demand and different holding cost function. [19] Teng, J. T. Krommyda, I. P., Scour, K., and Lou, K. R. a comprehensive extension of optimal ordering policy for stock dependent demand under progressive payment scheme. [20] Urban T. L An inventory model with an inventory-level dependent demand rate and relaxed terminal conditions. [21] Wang, Y. and Gerchak, Y Supply chain coordination when demand is self-space dependent Manufacturing and service operations management. [22] Yang C. T. Ouyang, L. Y.Wu, K. S. and Yen, H. F An optimal replenishment policy for deteriorating items with stock-dependent demand power relaxed terminal conditions under limited storage space capacity.

Assumption:

1. Production rate is constant at any time.
2. Production starts when inventory level is zero and it stops when inventory level is highest.
3. Inventory level is highest at $\Theta=t_1$. Since the production stops while inventory is highest, inventory depletes quickly due to demand and decay.

4. Demand rate exponentially decreases.
5. Deteriorating or decay rate is constant and very small.
6. Decreasing rate of demand is also constant and less than decay rate for unit inventory.
7. Shortages are not allowed.
8. Lead time is zero.

Development of the Model: The model is developed on the basis of exponential market demands and constant production capacity of the organization. The model is suitable for the products which have finite shelf-life and ultimately causes the products decay. At the beginning i.e. at time Ɵ=0, the production starts with zero inventory. In this model, the production rate Ƥ remains constant for entire production cycle. But the demands exponentially decrease time to time, which is shown in the Figure.

During the time Ɵ=0 to t_1, the inventory decreases at the rate of Ƥ - $Q_0 e^{-\gamma\theta}$-μ R (Ɵ), as $Q_0 e^{-\gamma\theta}$ is the market demand. Here, μR (Ɵ) is the decay of R (Ɵ) inventories at instant Ɵ. By using the above arguments, we can have the following differential equation,

$$\frac{d}{d\theta}R(\Theta) + \mu R(\Theta) = Ƥ - Q_0 e^{-\gamma\theta}$$

The general solution of the differential equation is,

$$R(\Theta) = \frac{Ƥ}{\mu} - \frac{Q_0 e^{-\gamma\theta}}{\mu-\gamma} + Ae^{-\mu}$$

We now apply the following boundary condition, at Ɵ=0, we get R (Ɵ) =0

$$A = \frac{Q_0}{\mu-\gamma} - \frac{Ƥ}{\mu}$$, by solving we get,

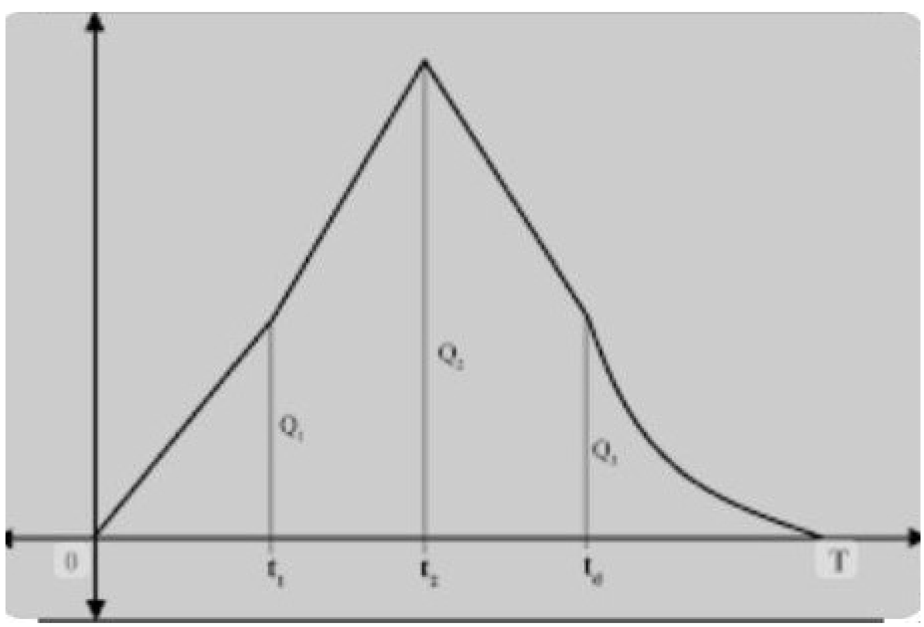

Figure 1: Time verses quantity.

Therefore,

$$R(\Theta) = \frac{Ƥ}{\mu}(1 - e^{-\mu\theta}) + \frac{Q_0}{\mu-Ƥ}(e^{-\mu\theta} - e^{-\gamma\theta}) \quad \ldots (1)$$

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Applying the other boundary condition, we get

R (Θ) = Q_1 at Θ = t_1 . Then Equation (1) gives us,

$Q_1 = \frac{p}{\mu}(1- e^{-\mu t_1}) + \frac{Q_0}{\mu-\gamma}(e^{-\mu t_1} - e^{-y t_1})$ … (2)

Hence, the total Un – decayed inventory during Θ = 0 to t_1,

$R_1 = \int_0^{t_1} R(\theta)d\theta$

$= \int_0^{t_1}[\frac{p}{\mu}(1- e^{-\mu}\quad) + \frac{Q_0}{\mu-\gamma}(e^{-\mu\theta} - e^{-\gamma\theta})]$ dΘ

$=\frac{p}{\mu}[t_1 + \frac{e^{-\mu t_1}-1}{\mu}] - \frac{Q_0}{\mu-\gamma}[\frac{e^{-\mu t_1}-1}{\mu} - \frac{e^{-\gamma t_1}-1}{\gamma}]$

$= \frac{p}{\mu}t_1 + \frac{p}{\mu^2}[e^{-\mu t_1} - 1] - \frac{Q_0}{\mu(\mu-\gamma)}(e^{-\mu t_1}) + \frac{Q_0}{\gamma(\mu-\gamma)}(e^{-\gamma t_1} - 1)$

$= \frac{p}{\mu}t_1 + \frac{p}{\mu^2}(-\mu t_1 + \frac{\mu^2 t_1{}^2}{2}) - \frac{Q_0}{\mu(\mu-\gamma)}(-\mu t_1 + \frac{\mu^2 t_1^2}{2}) + \frac{Q_0}{\gamma(\mu-\gamma)}(-\gamma t_1 + \frac{\gamma^2 t_1^2}{2})$

Neglecting the higher power of μ and ɤ, it becomes,

$=\frac{p}{\mu}t_1 - \frac{p}{\mu}t_1 + \frac{p}{2}t_1 + \frac{Q_0}{\mu-\gamma}t_1 - \frac{Q_{0\mu}}{2(\mu-\gamma)}t_1^2 - \frac{Q_0}{\mu-\gamma}t_1 + \frac{Q_{0\gamma}}{2(\mu-\gamma)}t_1^2$

$=[\frac{p}{2} - \frac{Q_{0\mu}}{2(\mu-\gamma)} + \frac{Q_{0\gamma}}{2(\mu-\gamma)}]t_1^2$

$= \frac{1}{2}(p - Q_0)t_1^2$ … (3)

Again, during Θ = t_1 to T, the inventory decreases at the of $Q_0 e^{-\gamma\theta}$. Thereby, following the process of forming first differential equation, we get the second differential equation as below:

$\frac{d}{d\theta}$R (Θ) + μ R (Θ) = - $Q_0 e^{-\gamma\theta}$.

Applying the boundary condition at Θ = T, we Consider, R (Θ) =. Then we get the general solution as below:

R (Θ) = $-\frac{Q_0 e^{-\gamma\theta}}{\mu-\gamma} + \frac{Q_0 e^{(\mu-\gamma)T-\mu\theta}}{\mu-\gamma}$ … (4)

We get the following conditions, If we put the other boundary condition in equation no (4), i.e. Θ =t_1, R (Θ) = Q_1

$Q_1 = \frac{Q_0}{\mu-\gamma}e^{(\mu-\gamma)T-\mu t_1} - \frac{Q_0}{\mu-\gamma}e^{-\gamma t_1}$ … (5)

Hence, with the help of equation no (4), we get the un – decayed inventory during Θ=t_1 to T,

$R_2 = \int_{t_1}^{T} R(\theta)d(\theta)$

$=\int_{t_1}^{T}[-\frac{Q_0 e^{-\gamma\theta}}{\mu-\gamma} + \frac{Q_0 e^{(\mu-\gamma)T-\mu\theta}}{\mu-\gamma}]$dΘ

$= \frac{Q_0}{\mu-\gamma}[\frac{e^{-\gamma T}-e^{-\gamma}t_1}{\gamma}] - \frac{Q_0}{\mu-\gamma}[\frac{e^{(\mu-\gamma)T-\mu T}-e^{(\mu-\gamma)T-\mu t_1}}{\mu}]$

$= \frac{Q_0}{\gamma(\mu-\gamma)}(e^{-\gamma T} - e^{-\gamma}t_1) - \frac{Q_0}{\gamma(\mu-\gamma)}(e^{-\gamma T} - e^{(\mu-\gamma)T-\mu t_1})$

$= \frac{Q_0}{\gamma(\mu-\gamma)}e^{(\mu-\gamma)T-\mu t_1} + \frac{Q_0}{\gamma\mu}e^{-\gamma T} - \frac{Q_0}{\gamma(\mu-\gamma)}e^{-\gamma t_1}$

$= \frac{Q_0}{\gamma(\mu-\gamma)}[1+ (\mu-\gamma)T - \mu t_1 + \frac{1}{2}\{(\mu-\gamma)T - \mu t_1\}^2] + \frac{Q_0}{\gamma\mu}[1 - \gamma T + \frac{1}{2}\gamma^2 T^2] - \frac{Q_0}{\mu(\mu-\gamma)}[1 - \gamma t_1 + \frac{1}{2}\gamma^2 t_1]$

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$= \frac{Q_0}{\gamma(\mu-\gamma)} + \frac{Q_0}{\mu}T - \frac{Q_0}{(\mu-\gamma)}t_1 + \frac{Q_{0(\mu-\gamma)}}{2\mu}T^2 + \frac{Q_0\mu}{2(\mu-\gamma)}t_1^2 - Q_0Tt_1 + \frac{Q_0}{\mu\gamma} - \frac{Q_0}{\mu}T + \frac{Q_{0\gamma}}{2\mu}T^2 - \frac{Q_)}{\gamma(\mu-\gamma)} + \frac{Q_0}{\mu-\gamma}t_1 - \frac{Q_{0\mu}}{2(\mu-\gamma)}t_1^2$

$= -\frac{Q_0}{\mu\gamma} + \frac{Q_{0\mu}}{2\mu}T^2 + \frac{Q_0}{2}t_1^2 - Q_0Tt_1 + \frac{Q_0}{\mu\gamma}$

$= \frac{1}{2}Q_0(T - t_1)^2$ … (6)

And, the deteriorating items during $\Theta = t_1$ to T,

$D_2 = Q_1 - \int_{t_1}^{T} R(\Theta)d\Theta = Q_1 - \int_{t_1}^{T} e^{-\Theta\gamma}d\Theta$

$= \frac{Q_0}{\mu-\gamma}e^{(\mu-\gamma)T-\mu t_1} - \frac{Q_0}{\mu-\gamma}e^{-\gamma t_1} + \frac{Q_0}{\gamma}(e^{-\gamma T} - e^{-\gamma}t_1)$

Neglecting the higher power of μ and ɤ , it becomes,

$= \frac{Q_0}{\mu-\gamma}[1 + (\mu-\gamma)T - \mu t_1 + \frac{1}{2}\{(\mu-\gamma)T - \mu t_1\}^2] - \frac{Q_{0\mu}}{\gamma(\mu-\gamma)}[1-\gamma t_1 + \frac{1}{2}\gamma^2t_1^2] + \frac{Q_0}{\gamma}[1-\gamma T + \frac{1}{2}\gamma^2T^2]$

$= \frac{Q_0}{\mu-\gamma} + Q_0T - \frac{Q_{0\mu}}{\mu-\gamma}t_1 + \frac{Q_0T^2(\mu-\gamma)}{2} + \frac{Q_0\mu^2t_1^2}{2(\mu-\gamma)} - Q_0\mu Tt_1 - \frac{Q_0\mu}{\gamma(\mu-\gamma)} + \frac{Q_0\mu}{\mu-\gamma} - \frac{Q_0\mu\gamma t_1^2}{2(\mu-\gamma)} + \frac{Q_0}{\gamma} - Q_0T + \frac{Q_0\gamma T^2}{2}$

$= \frac{Q_0}{\mu-\gamma} - \frac{Q_{0\mu}}{\gamma(\mu-\gamma)} + \frac{Q_0}{\gamma} + [\frac{Q_0(\mu-\gamma)}{2} + \frac{Q_0}{2}]T^2 + [\frac{Q_0\mu^2}{2(\mu-\gamma)} - \frac{Q_0\mu\gamma}{2(\mu-\gamma)}]t_1^2 - Q$

$= \frac{Q_0\mu}{2}T^2 + \frac{Q_0\mu}{2}t_1^2 - Q_0\mu Tt_1$

$= \frac{Q_0\mu}{2}T^2 + \frac{Q_0\mu}{2}t_1^2 - Q_0\mu Tt_1$

$= \frac{Q_0\mu}{2}(T - t_1)^2$ … (7)

Total cost function: Total average inventory cost per unit time per cycle can be expressed as below,

$TC(t_1, T) = \frac{K + hR_1 + (R_2 + D_2)}{T}$

By using the equation (3), (6) and (7), Now we get,

$TC(t_1, T) = \frac{K}{T} + \frac{h}{T}[\frac{Þ-Q_0}{2}t_1^2 + \frac{Q_0(T-t_1)^2}{2}] + \frac{\eta}{T}[\frac{Q_0\mu(T-t_1)^2}{2}]$ …(8)

Now, the objective is to minimize the total inventory cost TC. For the minimum average inventory cost TC the optimum values of time t_1 and T are the solution of the following convex property:

$\frac{\partial}{\partial t_1}TC(t_1, T) = 0 \quad And \quad \frac{\partial}{\partial T}TC(t_1, T) = 0$ … (A)

$(\frac{\partial^2 TC}{\partial t_1})\left(\frac{\partial^2 TC}{\partial T_1}\right) - (\frac{\partial^2 TC}{\partial t_1 \partial T})^2 > 0$ … (B)

The cost function will be convex if these well recognized criteria are satisfied. Thereby, we can determine the total optimum cost TC, Optimum time interval t_1 total time cycle T^* and the optimum order quantityQ^*. Now,

$\frac{\partial TC}{\partial t_1} = \frac{h}{T}[(Þ - Q_0)t_1 - Q_0(T - t_1)] - \frac{\eta}{T}[Q_0\mu(T - t_1)]$ … (9)

$\frac{\partial^2 TC}{\partial t_1^2} = \frac{hÞ}{T} + \frac{\eta\, Q_{0\mu}}{T}$

$= \frac{1}{T}(hÞ + \eta Q_0\mu)$ … (10)

$\frac{\partial^2 TC}{\partial T \partial t_1} = \frac{h}{T}(-Q_0) - \frac{h}{T^2}(Þt_1 + Q_0 - Q_0T) - \frac{\eta}{T}(Q_0\mu) + \frac{\eta}{T^2}(Q_0\mu)(T - t_1)$

$= \frac{-h}{T}(Q_0) - \frac{h(Ɗt_1+Q_0)}{T^2} + \frac{hQ_0}{T} - \frac{\eta Q_0\mu}{T} + \frac{\eta Q_0\mu}{T} - \frac{\eta Q_0\mu t_1}{T^2}$

$= -\frac{1}{T^2}(hQ_0 + hƊt_1 + \eta Q_0\mu t_1)$

$= -\frac{1}{T^2}[hQ_0 + (hƊ + \eta Q_0\mu)t_1]$ …(11)

$\frac{\partial TC}{\partial T} = \frac{-K}{T^2} + \frac{h}{T}[Q_0(T-t_1)] - \frac{h}{T^2}[\frac{(Ɗ-Q_0)t_1^2}{2} + Q_0t_1 + \frac{Q_0(T-t_1)^2}{2}] - \frac{\eta}{T^2}[\frac{Q_0(T-t_1)^2}{2}] + \frac{\eta}{T}[hQ_0\mu(T-t_1)]$

$= \frac{-K}{T^2} + hQ_0 - \frac{h}{T}Q_0t_1 - \frac{h(Ɗ-Q_0)t_1^2}{2T^2} - \frac{hQ_0t_1}{T^2} - \frac{hQ_0(T-t_1)^2}{2T^2} - \frac{\eta Q_0\mu(T-t_1)^2}{2T^2} + \eta Q_0\mu - \frac{\eta Q_0\mu t_1}{T}$ …(12)

$\frac{\partial^2 TC}{\partial T^2} = \frac{2K}{T^3} + \frac{hQ_0t_1}{T^2} + \frac{h(Ɗ-Q_0)t_1^2}{T^3} + \frac{2hQ_0t_1}{T^3} + \frac{hQ_0(T-t_1)^2}{T^3} - \frac{hQ_0(T-t_1)}{T^2} + \frac{\eta Q_0\mu(T-t_1)^2}{T^3} - \frac{\eta Q_0\mu t_1}{T^2}$

$= \frac{2K}{T^3} + \frac{hQ_0t_1}{T^2} + \frac{hƊt_1^2}{t^3} - \frac{hQ_0t_1^2}{T^3} + \frac{2hQ_0t_1}{T^3} - \frac{hQ_0(T-t_1)t_1}{T^3} - \frac{\eta Q_0\mu(T-t_1)t_1}{T^3}$

$= \frac{2K}{T^3} + \frac{hƊt_1^2}{T^3}\frac{\eta Q_0t_1^2}{T^3} + \frac{2hQ_0t_1}{T^3}$ … (13)

Now using the equations (10), (11), (13), we get,

$(\frac{\partial^2 TC}{\partial t_1^2})(\frac{\partial^2 TC}{\partial T^2}) - (\frac{\partial^2 TC}{\partial t_1 \partial T})^2$

$= \frac{1}{T}(hƊ + \eta Q_0\mu)[\frac{2K}{T^3} + \frac{hƊt_1^2}{T^3} + \frac{\eta Q_0\mu t_1^2}{T^3} + \frac{2hQ_0t_1}{T^3}] - \frac{1}{T^4}(hQ_0 + hƊt_1 + \eta Q_0\mu t_1)^2$

$= \frac{1}{T^4}(2KhƊ + h^2Ɗ^2t_1^2 + h\eta ƊQ_0\mu t_1^2 + 2h^2ƊQ_0t_1 + 2K\eta Q_0\mu + h\eta Q_0\mu t_1^2 + \eta^2{Q_0}^2\mu^2t_1^2)$ +

$\frac{1}{T^4}(2h\eta{Q_0}^2\mu t_1 - h^2Q_0^2 - h^2Ɗ^2t_1^2 - \eta^2Q_0^2\mu^2t_1^2 - 2h^2\eta Q_0t_1 - 2h\eta Q_0^2\mu t_1 - 2h\eta ƊQ_0\mu t_1^2)$

$= \frac{1}{T^4}(2KhƊ + 2K\eta Q_0\mu - h^2Q_0^2)$

This term will be greater than zero, i.e. convex property (A) will we satisfied, if $2KhƊ+2K\eta Q_0\mu - h^2Q_0^2$ is greater than zero.

Hence, $2KhƊ+2K\eta Q_0\mu - h^2Q_0^2 > 0$ is to be satisfied for the minimum optimum cost. Now putting the values 0f (9) and (12) in the convex property (A), i.e.

$\frac{\partial}{\partial t_1}TC(t_1, T) = 0$ and $\frac{\partial}{\partial T}TC(t_1, T) = 0$ respectively and thereby we get the optimum time interval.

Form the equation $\frac{\partial}{\partial t_1}TC = 0$, we get the value of T as below,

$T = \frac{hQ_0 + (hƊ + \eta Q_0\mu)t_1}{Q_0(h+\eta\mu)}$ … (14)

Again from $\frac{\partial}{\partial T}TC = 0$, the value of T can be obtained as follows,

$T = \pm\sqrt{\frac{hƊ + \eta Q_0\mu)t_1^2 + 2hQ_0t_1 + 2K}{Q_0(h+\eta\eta)}}$ … (15)

Now from the equation no (9) and (10) with the positive value of T from (10), we get,

$\frac{hQ_0 + (hƊ + \eta Q_0\mu)t_1}{Q_0(h+\eta\mu)} = \sqrt{\frac{hƊ + \eta Q_0\mu)t_1^2 + 2hQ_0t_1 + 2K}{Q_0(h+\eta\eta)}}$

Solving this equation we now get the value of t_1 which is mentioned here,

$t_1 = \frac{-2h^2Q_0(Ɗ-Q_0) \pm \sqrt{4h^2Q_0^2(Ɗ-Q_0)^2 - 4hQ_0(Ɗ-Q_0)(hƊ+\eta Q_0\mu)(h^2Q_0-2h)}}{2h(Ɗ-Q_0)(hƊ+\eta Q_0\mu)}$

$$= \frac{-2h^2Q_0(\text{Ꝑ}-Q_0) \pm \sqrt{4h^2Q_0^2(\text{Ꝑ}-Q_0)^2-4hQ_0(\text{Ꝑ}-Q_0)(h\text{Ꝑ}+\eta Q_0\mu)(2\eta K\mu+ \ )}}{2h(\text{Ꝑ}-Q_0)(h\text{Ꝑ}+\eta Q_0\mu)} \quad \ldots (16)$$

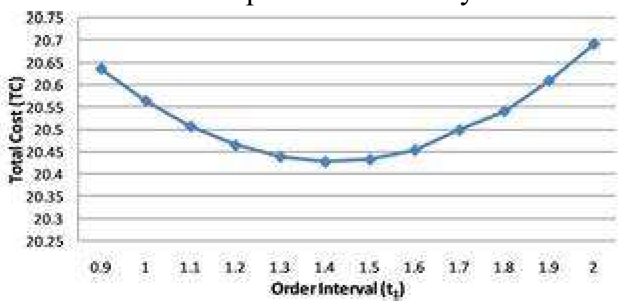
Illustration: Here, we provided a numerical illustration to justify the optimum inventory cost and the optimum order cycle.

Let us consider, the inventory system has the following parameters,

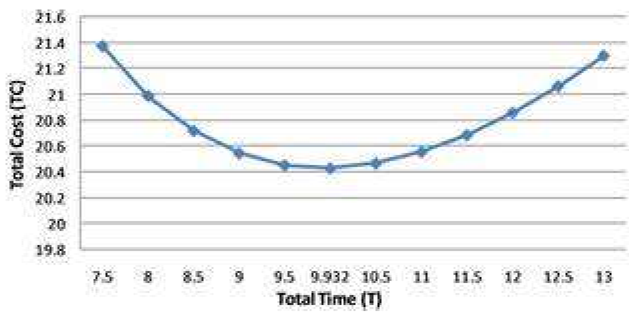
ϒ = 0.1; K = 100, h = 1; η = 0.5; Ꝑ = 15; Q_0 = 2; μ = 0.4;

2Khη + 2KηQ_0μ - $h^2Q_0^2$ = 3076 > 0

These parameters we check the condition, Now, putting all the values in equation no (5), (16), (14) and (8) we get the optimum order quantity Q_1^* = 68.62 units, optimum order interval t_1^* = 1.418, total time cycle T^* = 9.932 units and total optimum inventory cost TC^* = 20.428 units.



Order interval verses total cost



Total Time verses total cost

Consider an inventory system with the following parametric values in proper units:

P = 2000, α = 2.0, β = 1.5, γ = 10, η = 50, a = 0.15, b = 1.5, c = 0.10. The output of the program by using maple mathematical software is T = 1.4221, t_1 = 0.01609, and T_c = 70.302357the value of t_1 at which the inventory level become zero is 0.016. The effect of changes in the parameter of the inventory model is as follows.

Parameter	% Change	T	t_1	T_c
P	+40	1.41997	0.01167	70.41517
	+20	1.42145	0.01469	70.33800
	-20	1.42400	0.01989	70.20544
	-40	1.42702	0.02607	70.04716
a	+40	1.42187	0.01563	70.31200
	+20	1.42200	0.01568	70.31200

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	-20	1.42228	0.01634	70.29726
	-40	1.42244	0.01660	70.29196
b	+40	1.42285	0.17321	70.27687
	+20	1.42267	0.01701	70.28255
	-20	1.42089	0.01362	70.36365
	-40	1.41854	0.00842	70.50600
c	+40	1.42217	0.01615	70.30124
	+20	1.42215	0.01612	70.30180
	-20	1.42212	0.01607	70.30291
	-40	1.42211	0.01604	70.30346
α	+40	1.42011	0.01190	70.44910
	+20	1.42098	0.01136	70.37548
	-20	1.42375	0.01946	70.22971
	-40	1.42610	0.02450	70.15754
B	+40	1.42212	0.01606	70.30295
	+20	1.42213	0.01607	70.30265
	-20	1.42215	0.01611	70.30206
	-40	1.42216	0.01611	70.30178

The above numerical illustration shows that the model is quite stable by changing in the parameter of the model.

Conclusion And Future Scope: - A deterministic account model-based request reliant on time elevated with constant deterioration amount, and constant holding fee is erected in this paper. The total optimum cost has been considered for the ethics of Ω and T, sufficient the necessary condition. The model has been verified using mathematical and graphical diagrams. We have accessible the mathematical models above two dissimilar cases are also deliberated which can help the conclusion brands to regulate the total normal inventory cast. To the customer to earn more by conceiving the resource else after the sale-proceed of the account, which consequences in the inferior cost. This model mentions to the model in which request alterations slightly over time as product charges increase gradually. This model is more accurate to the atmosphere. As a result, future metropolises and businesses will benefit greatly from the paradigm.

This paper develops a periodic review EOQ model with inventory-induced demand under deterioration and cash discount. The period of cash discount is considered to be shorter than the period of permissible delay. Four different cases have been discussed. The optimal solutions are obtained by using second order approximation and differential calculus. The main objective of this study is to minimize the total cost. Through numerical study and sensitivity analysis, it is seen that high discount rate results in slight increase in order quantity and decrease in total cost. This paper may be further generalized for allowing shortages. We could also extend the model by adding freight charges and advertisement costs.

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