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NEW CLASS OF GENERALIZED-OPEN SETS IN IDEAL $\mathcal{T}$-TOPOLOGICAL SPACES

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Abstract: The concept of open and closed sets is one of the most powerful tools for the study of digital topology and computer graphics in the modern scientific world. In the present paper, we introduce and investigate the notions of several $\mathcal{T}$-open sets such as $\mathcal{T}_{\mathfrak{X}}$-open of these sets by making the use of some suitable and counter examples for the justification of results. Hence we have categorized a new class of generalized open sets in this paper.

Keywords: *$\mathcal{T}$-open, $\mathcal{T}$-semi-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-α-open, $\mathcal{T}$-β-open, $\mathcal{T}_{\mathfrak{X}}$-open*
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1. Introduction

.Veera Kumar [29-30] studied the concepts of closed sets, g-closed sets and $g^{\#}$-closed sets. Further the authors [26] verified various results and provide certain characterization of closed sets in topological spaces.

The subject of ideals in topological spaces was studied by Kuratowski [16] almost half a century ago, which motivated the research in applying topological ideals to generalize the most basic properties in general topology. Jankovic and Hamlett [14-15] introduced the concept of I-open sets in ideal topological spaces that initialized the application of topological ideals in the generalization of most fundamental properties in general topology provide the concept of compactable extensions of ideals extend the concept of topologies from old via ideals. Abd El. Monsef [1-3], Dontchev [10] and Yuksel [31-32] provide some major contributions in the field. They introduced and studied several generalized closed and open sets in ideal topological spaces such as I-closed sets, pre-I-open sets, semi-I-open sets and alpha-I-open sets. The authors [11-12, 27-28] studied all the concepts related to the decomposition of continuity, complete continuity and almost-I-continuity.

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Further the authors discussed some $\beta -$ I-open sets. Jafari1 and Rajesh [13] studied some concepts related to generalize closed sets in an ideal topological space and obtained some characterizations. Egenhofer [9] discussed the very useful concept for binary topological relations. Gevorgyan [10] studied the group of continuous binary operations on a topological space and established its relationship with the group of homeomorphisms. Chen et al. [4] demonstrated the dynamics on binary relations over topological spaces. Nithyanantha and Thangavelu [20] studied the concept of binary topology and investigated some of its basic properties.

In this paper, we developed the very useful concept of $\mathcal{T}_{\mathfrak{I}}$*-open* and established the relationship of these sets with some other generalized sets. The results have been shown by several counter examples and applications. Some require basic definitions, concepts of $\mathcal{T}$*-topological*, *ideal* $\mathcal{T}$*-topological spaces* and notations are discussed in Section 2. $\mathcal{T}_{\mathfrak{I}}$*-open sets* are discussed in Section 3. The applications with future scope of paper are discussed in Section 5. Finally, in Section 6 concludes the paper.

2. Preliminaries

Definition 2.1: If $\mathcal{Z}$ is a non-empty set and the $\mathcal{T}$ a collection of subsets of $\mathcal{Z}$ satisfying axioms i.e., $\emptyset$ and entire set $\mathcal{Z}$ are in $\mathcal{T}$ and the union of the elements of any sub collection of $\mathcal{T}$ is in $\mathcal{T}$. Then $\mathcal{T}$ is said to be GT on $\mathcal{Z}$ and set the $\mathcal{Z}$ together with the topology $\mathcal{T}$ on $\mathcal{Z}$ is known as GTS, and is denoted by $(\mathcal{Z},\mathcal{T})$. The elements of $\mathcal{T}$ are called $\mathcal{T}$-open sets and their complements are called as $\mathcal{T}$-closed sets.

Example 2.1: If $\mathcal{Z} = \{1,2,3\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1,2\}, \{1,3\}, \mathrm{Z}\}$ is a GT on $\mathcal{Z}$.

Definition 2.2: If $(\mathcal{Z},\mathcal{T})$ is a GTS and $\mathrm{S} \subseteq \mathcal{Z}$. Then the union of all $\mathcal{T}$-open sets in $\mathcal{Z}$ contained in S is called $\mathcal{T}$-interior of S and is denoted by $\mathfrak{I}_{\mathcal{T}}(\mathrm{S})$.

Definition 2.3: If $(\mathcal{Z},\mathcal{T})$ is a GTS and $\mathrm{S} \subseteq \mathcal{Z}$. Then the intersection of all $\mathcal{T}$-closed sets in $\mathcal{Z}$ containing S is called $\mathcal{T}$-closure of S and is denoted by $\mathcal{C}_{\mathcal{T}}(\mathrm{S})$.

Definition 2.4: If $(\mathcal{Z},\mathcal{T})$ is a GTS and $\mathrm{S} \subseteq \mathcal{Z}$. Then S is called

i) $\mathcal{T}$-semi open if $\mathrm{S} \subseteq \mathcal{C}_{\mathcal{T}}(\mathfrak{I}_{\mathcal{T}}(\mathrm{S}))$

ii) $\mathcal{T}$-pre open if $\mathrm{S} \subseteq \mathfrak{I}_{\mathcal{T}}(\mathcal{C}_{\mathcal{T}}(\mathrm{S}))$

iii) $\mathcal{T}$-α open if $\mathrm{S} \subseteq \mathfrak{I}_{\mathcal{T}}(\mathcal{C}_{\mathcal{T}}(\mathfrak{I}_{\mathcal{T}}(\mathrm{S})))$

iv) $\mathcal{T}$-regular open if $\mathrm{S} = \mathfrak{I}_{\mathcal{T}}(\mathcal{C}_{\mathcal{T}}(\mathrm{S}))$

Proposition 2.1:

i) Each $\mathcal{T}$-open set is $\mathcal{T}$-α-open.

ii) Each $\mathcal{T}$-α open set is $\mathcal{T}$-semi-open.

iii) Each $\mathcal{T}$-α-open set is $\mathcal{T}$-pre-open.

iv) Each $\mathcal{T}$-regular open set is $\mathcal{T}$-open.

Remark 2.1: The converse of the Proposition 2.1 is not true in general which can be seen in Example 2.2, Example 2.3, Example 2.4 and Example 2.5.

Example 2.2: If $\mathcal{Z} = \{1, 2, 3, 4\}$ and $\mathcal{T} = \{\emptyset, \{2,3\}, \{1,3, 4\}, \mathcal{Z}\}$ is GTS. Then the set $\{1,2, 3\}$ is $\mathcal{T}$-α-open but not $\mathcal{T}$-open set.

Example 2.3: If $\mathcal{Z} = \{1, 2, 3, 4\}$ and $\mathcal{T} = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{2,3,4\}, Z\}$ is GT. Then the set $\{1,2\}$ is $\mathcal{T}$-semi-open but not $\mathcal{T}$-α-open.

Example 2.4: If $\mathcal{Z} = \{1, 2, 3, 4\}$ and $\mathcal{T} = \{\emptyset, \{1,2\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{2,3,4\}, \mathcal{Z}\}$ is GT. Then the set $\{1,3,4\}$ is $\mathcal{T}$-pre-open but not $\mathcal{T}$-α-open.

Example 2.5: If $\mathcal{Z} = \{1, 2, 3, 4\}$ and $\mathcal{T} = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{2,3,4\}, \mathcal{Z}\}$ is GT. Then the set $\{2,3\}$ is $\mathcal{T}$-open but not $\mathcal{T}$-regular-open.

Definition 2.5: If $\mathcal{Z}$ is non-empty set. Then an ideal is a non-empty collection $\mathfrak{T}$ of subsets of $\mathcal{Z}$ satisfying the axioms i.e. if. $P \in \mathfrak{T}$ and $Q \subseteq P$ implies $Q \in \mathfrak{T}$ and If $P \in \mathfrak{T}$ and $Q \in \mathfrak{T}$ implies $(P \cup Q) \in \mathfrak{T}$. If $(\mathcal{Z}, \mathcal{T})$ is GTS and $\mathfrak{T}$ ideal of subsets of $\mathcal{Z}$, then $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS.

Example 2.6: If $(\mathcal{Z}, \mathcal{T})$ is GTS. Then the collection $\mathfrak{T} = \{\emptyset\}$ and $\mathfrak{T} = \wp(\mathcal{Z})$ are also ideals on $\mathcal{Z}$.

Definition 2.6: If $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS and $P \subseteq \mathcal{Z}$. Then the set $(P)^*(\mathfrak{T}) = \{ x \in \mathcal{Z} / (S \cap P) \notin \mathfrak{T}$ for each neighborhood S of x$\}$ is called the local function of P in respect of $\mathfrak{T}$ and write P^* instead of $(P)^*(\mathfrak{T})$ to avoid confusion.

3. $\mathcal{T}_{\mathfrak{T}}$-open Sets

Definition 3.1: If $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS and $P \subseteq \mathcal{Z}$, then P is said to be **$\mathcal{T}_{\mathfrak{T}}$-semi-open** if there is any $\mathcal{T}$-open set S such that $P - \mathcal{C}_{\mathcal{T}}(S) \in \mathfrak{T}$ and $S - P \in \mathfrak{T}$.

Example 3.1: If $\mathcal{Z} = \{1,2,3\}$, $\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2\}, \{3\}, \{2,3\}\}$. Clearly $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS. Therefore the set $P = \{1\}$ is $\mathcal{T}_{\mathfrak{T}}$-semi-open, because for $S = \{1, 2\}$, $P - \mathcal{C}_{\mathcal{T}}(S) = \{1\} - \mathcal{Z} = \emptyset \in \mathfrak{T}$ and $(\{1, 2\} - \{1\}) = \{2\} \in \mathfrak{T}$.

Remark 3.1: Each $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set is $\mathcal{T}_{\mathfrak{T}}$-semi-open. The converse part is not true which can be seen in Example 3.2.

Example 3.2: If$\mathcal{Z} = \{1, 2, 3\}$, $\mathcal{T} = \{\emptyset, \{1, 3\}, \{2, 3\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{3\}, \{2,3\}\}$. Therefore $\{1\}$ is $\mathcal{T}_{\mathfrak{T}}$-semi-open in $(\mathcal{Z}, \mathcal{T})$ but not $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set.

Remark 3.2: Each $\mathcal{T}_{\mathfrak{T}}$- α-open is $\mathcal{T}_{\mathfrak{T}}$-semi-open set. The converse part is not true which can be seen in Example 3.3.

Example 3.3: If $\mathcal{Z} = \{1, 2, 3,4\}$, $\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}. \{1, 3\}, \{1, 4\}, \{2, 3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{1,2,4\}, \{2,3,4\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2\}\}$. Therefore the set $\{1,3,4\}$ is $\mathcal{T}_{\mathfrak{T}}$-semi-open but not $\mathcal{T}_{\mathfrak{T}}$- α-open set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Definition 3.2: If $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS and $P \subseteq \mathcal{Z}$, then P is said to be **$\mathcal{T}_{\mathfrak{T}}$-α-open** if there is any $\mathcal{T}$-open set S such that $P - \mathfrak{T}_{\mathcal{T}}(\mathcal{C}_{\mathcal{T}}(S)) \in \mathfrak{T}$ and $S - P \in \mathfrak{T}$.

Example 3.4: The set $P = \{1\}$ in Example 3.1 is $\mathcal{T}_{\mathfrak{T}}$-α-open set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.3: Each $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set is $\mathcal{T}_{\mathfrak{T}}$-α-open. The converse part is not true which can be seen in Example 3.2.

Definition 3.3: If $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS and $P \subseteq \mathcal{Z}$, then P is said to be **$\mathcal{T}_{\mathfrak{T}}$-pre-open** if there is any $\mathcal{T}$-open set S such that $S - \mathcal{C}_{\mathcal{T}}(P) \in \mathfrak{T}$ and $P - S \in \mathfrak{T}$.

Example 3.5: If $\mathcal{Z} = \{1, 2, 3\}$, $\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2\}, \{2.3\}\}$. Therefore the $\{3\}$ is $\mathcal{T}_{\mathfrak{T}}$-pre-open set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.4: Each $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set is $\mathcal{T}_{\mathfrak{T}}$-pre-open. The converse part is not true which can be seen in Example 3.6.

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Example 3.6: The set $\{3\}$ is $\mathcal{T}_{\mathfrak{T}}$-pre-open set in Example 3.5 but not $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Definition 3.4: If $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$ is GITS and P $\subseteq \mathcal{Z}$, then P is said to be $\boldsymbol{\mathcal{T}_{\mathfrak{T}}}$**-β-open** if there is any $\mathcal{T}$-open set S such that $\mathrm{S} - \mathfrak{T}_{\mathcal{T}}(\mathcal{C}_{\mathcal{T}}(\mathrm{P})) \in \mathfrak{T}$ and $\mathrm{P} - \mathrm{S} \in \mathfrak{T}$.

Example 3.7: The set $\{3\}$ in Example 3.5 is $\mathcal{T}_{\mathfrak{T}}$-β-open set in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.5: Each $\mathcal{T}$-open ($\mathcal{T}$-semi-open, $\mathcal{T}$-α-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-β-open) set is $\mathcal{T}_{\mathfrak{T}}$-β-open. The converse part is not true which can be seen in Example 3.5.

Remark 3.6: Each $\mathcal{T}_{\mathfrak{T}}$-β-open set is $\mathcal{T}_{\mathfrak{T}}$-pre-open. The converse part is not true which can be seen in Example 3.8.

Example 3.8: If $\mathcal{Z} = \{1, 2, 3\}$, $\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, \mathcal{Z}\}$ and $\mathfrak{T} = \{\emptyset, \{2\}\}$. Therefore the set $\{3\}$ is $\mathcal{T}_{\mathfrak{T}}$-pre-open set in $(\mathcal{Z}, \mathcal{T})$ but not $\mathcal{T}_{\mathfrak{T}}$-β-open in $(\mathcal{Z}, \mathcal{T}, \mathfrak{T})$.

Remark 3.7: The following implications as shown in figure-1 are the direct consequences of definitions of $\mathcal{T}_{\mathfrak{T}}$-open sets that we discussed in this section.

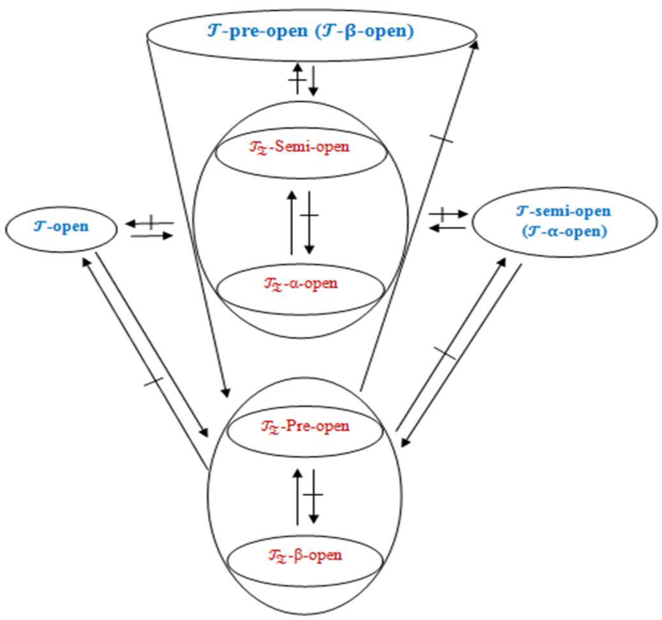

Figure-1: Relationships of $\mathcal{T}_{\mathfrak{T}}$-open sets

4. Applications with Future Scope

This concept of $\mathcal{T}$-open sets discussed in this paper can expected a very useful tool in the study of circuit theory. The sets discussed here are defined on the basis of the relationships i.e. the set is defined on the basis of two spaces such as generalized topological space and ideal topological space which makes the correct justification of categorization of sets.

Since the topology is playing a key role in almost every field of applied sciences and several branches of mathematics. Likewise the most precious and valuable topic in the mathematics is continuity. Thus the authors have provided lot of space for the different type's continuities which can be studied on the basis of these $\mathcal{T}$-open sets in general topology as discussed below:

Definition 6.1: If $(\mathcal{Z},\mathcal{T}_1)$ and $(\mathcal{Z},\mathcal{T}_2)$ are two GTS on $\mathcal{Z}$ and let $(\mathcal{Z},\mathcal{T}_1,\mathfrak{T})$ be an ideal $\mathcal{T}_1$-topological space. Then the mapping $\mathcal{F}:(\mathcal{Z},\mathcal{T}_1)\rightarrow(\mathcal{Z},\mathcal{T}_2)$ is said to be $\mathcal{T}_{\mathfrak{T}}$-semi-continuous map if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}_{\mathfrak{T}}$-semi-open in $(\mathcal{Z},\mathcal{T}_1)$ for every $\mathcal{T}_2$-open set (L, M) in $(\mathcal{Z},\mathcal{T}_2)$.

Example 6.1: If $\mathcal{Z}=\{1,2,3\}$. Then clearly $\mathcal{T}_1=\{\emptyset,\{1,3\},\{2,3\},\mathcal{Z}\}$ is $\mathcal{T}_1$-topology on $\mathcal{Z}$, $\mathcal{T}_2=\{\emptyset,\{1,2\},\{2,3\},\mathcal{Z}\}$ is $\mathcal{T}_2$-topology on $\mathcal{Z}$, and $\mathfrak{T}=\{\emptyset,\{3\},\{2,3\}\}$ is an ideal on $\mathcal{Z}$. Consider the mapping $\mathcal{F}:(\mathcal{Z},\mathcal{T}_1)\rightarrow(\mathcal{Z},\mathcal{T}_2)$ defined by $\mathcal{F}(1)=(1,2)$. Thus the inverse image of every $\mathcal{T}_2$-open set in $(\mathcal{Z},\mathcal{T}_2)$ is $\mathcal{T}_{\mathfrak{T}}$-semi-open in $(\mathcal{Z},\mathcal{T}_1)$. Hence $\mathcal{F}$ is $\mathcal{T}_{\mathfrak{T}}$-semi-continuous map.

Definition 6.2: If $(\mathcal{Z},\mathcal{T}_1)$ and $(\mathcal{Z},\mathcal{T}_2)$ are two GTS on $\mathcal{Z}$ and let $(\mathcal{Z},\mathcal{T}_1,\mathfrak{T})$ be an ideal $\mathcal{T}_1$-topological space. Then the mapping $\mathcal{F}:(\mathcal{Z},\mathcal{T}_1)\rightarrow(\mathcal{Z},\mathcal{T}_2)$ is said to be $\mathcal{T}_{\mathfrak{T}}$-pre-continuous map if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}_{\mathfrak{T}}$-pre-open in $(\mathcal{Z},\mathcal{T}_1)$ for every $\mathcal{T}_2$-open set (L, M) in $(\mathcal{Z},\mathcal{T}_2)$.

Example 6.2: If $\mathcal{Z}=\{1,2,3\}$. Then clearly $\mathcal{T}_1=\{\emptyset,\{1,2\},\{2,3\},\mathcal{Z}\}$ is $\mathcal{T}_1$-topology on $\mathcal{Z}$, $\mathcal{T}_2=\{\emptyset,\{1,3\},\{2,3\},\mathcal{Z}\}$ is $\mathcal{T}_2$-topology on $\mathcal{Z}$, $\mathfrak{T}=\{\emptyset,\{2\}\}$ is an ideal on $\mathcal{Z}$. Consider the mapping $\mathcal{F}:(\mathcal{Z},\mathcal{T}_1)\rightarrow(\mathcal{Z},\mathcal{T}_2)$ defined by $\mathcal{F}(3)=(1,3)$. Thus the inverse image of every $\mathcal{T}_2$-open set in $(\mathcal{Z},\mathcal{T}_2)$ is $\mathcal{T}_{\mathfrak{T}}$-pre-open in $(\mathcal{Z},\mathcal{T}_1)$. Hence $\mathcal{F}$ is $\mathcal{T}_{\mathfrak{T}}$-pre-continuous map.

Definition 6.3: If $(\mathcal{Z},\mathcal{T}_1)$ and $(\mathcal{Z},\mathcal{T}_2)$ are two GTS on $\mathcal{Z}$ and let $(\mathcal{Z},\mathcal{T}_1,\mathfrak{T})$ be an ideal $\mathcal{T}_1$-topological space. Then the mapping $\mathcal{F}:(\mathcal{Z},\mathcal{T}_1)\rightarrow(\mathcal{Z},\mathcal{T}_2)$ is said to be ${\mathcal{T}^*}_{\mathfrak{T}}$-continuous map if $\mathcal{F}^{-1}(L,M)$ is ${\mathcal{T}^*}_{\mathfrak{T}}$-open in $(\mathcal{Z},\mathcal{T}_1)$ for every $\mathcal{T}_2$-open set (L, M) in $(\mathcal{Z},\mathcal{T}_2)$.

Example 6.3: If $\mathcal{Z}=\{1,2,3\}$. Then clearly $\mathcal{T}_1=\{\emptyset,\{1,2\},\{2,3\},\mathcal{Z}\}$ is $\mathcal{T}_1$-topology on $\mathcal{Z}$, $\mathcal{T}_2=\{\emptyset,\{1,3\},\{2,3\},\mathcal{Z}\}$ is $\mathcal{T}_2$-topology on $\mathcal{Z}$, $\mathfrak{T}=\{\emptyset,\{2\},\{3\},\{2,3\}\}$ is an ideal on $\mathcal{Z}$. Consider the mapping $\mathcal{F}:(\mathcal{Z},\mathcal{T}_1)\rightarrow(\mathcal{Z},\mathcal{T}_2)$ defined by $\mathcal{F}(1)=(1,3)$ and $\mathcal{F}(2)=(2,3)$. Thus the inverse image of every $\mathcal{T}_2$-open set in $(\mathcal{Z},\mathcal{T}_2)$ is ${\mathcal{T}^*}_{\mathfrak{T}}$-open in $(\mathcal{Z},\mathcal{T}_1)$. Hence $\mathcal{F}$ is ${\mathcal{T}^*}_{\mathfrak{T}}$-continuous map.

Authors believed that the study can be fruitful in the field of digital topology also and can be a useful tool in study of the digital images in digital topology. The construction of generalized form of these sets can provide space for the study of digital continuity and digital paths for future researchers in the digital topology.

5. Conclusion

We aimed to introduce the concept of $\mathcal{T}_{\mathfrak{T}}$-open and ${\mathcal{T}^*}_{\mathfrak{T}}$-open sets in this paper. Then we established the relationship between above discuses sets and several other sets like $\mathcal{T}$-open, $\mathcal{T}$-semi-open, $\mathcal{T}$-pre-open, $\mathcal{T}$-α-open, $\mathcal{T}$-β-open, $\mathcal{T}_{\mathfrak{T}}$-semi-open, $\mathcal{T}_{\mathfrak{T}}$-pre-open, $\mathcal{T}_{\mathfrak{T}}$-α-open, $\mathcal{T}_{\mathfrak{T}}$-β-open and ${\mathcal{T}^*}_{\mathfrak{T}}$-open sets etc. The results have been shown by several counter examples. Further the study has provided the certain classification of other special class of open set which are more useful in studying the behaviour of some other open and closed sets in generalized topological spaces and generalized ideal topological spaces. This can be expected that the sets can useful in digital topology and circuit system where the connections are connected on the basis of relationships.

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